Equations & Inequalities

① The standard format of a linear equation is,

$ax + by + c = 0$ where $a, b \neq 0$ at a time.

OR $y = mx + c$ where m = slope of the line

②

Linear equation	a	b	c
$5x + 13y + 8 = 0$	5	13	8
$133x - 18y + 2k - 18 = 0$	133	-18	$2k - 18$
$(2p+3)x - (18ky) + 339 = 93$	$2p+3$	$-18k$	$339 - 93$
$2x + 3y = 88$	2	3	-88
$17kx + 5x - 3y = 18k - 23$			
i.e. $(17k+5)x - 3y - 18k + 23 = 0$	$(17k+5)$	-3	$-18k+23$
$2x = 83$			
i.e. $2x + 0y - 83 = 0$	2	0	-83
$8y = -2k - 99$			
i.e. $0x + 8y + 2k + 99 = 0$	0	8	$2k+99$
$kx - 13y - 25y - 9y + t = 0$			
$kx + (-38-9)y + t = 0$	k	$-(38+9)$	t
$5x + 13y = 27x - 40y + 88$	22	-53	88
$\therefore -22x + 53y - 88 = 0$			
i.e. $22x - 53y + 88 = 0$			

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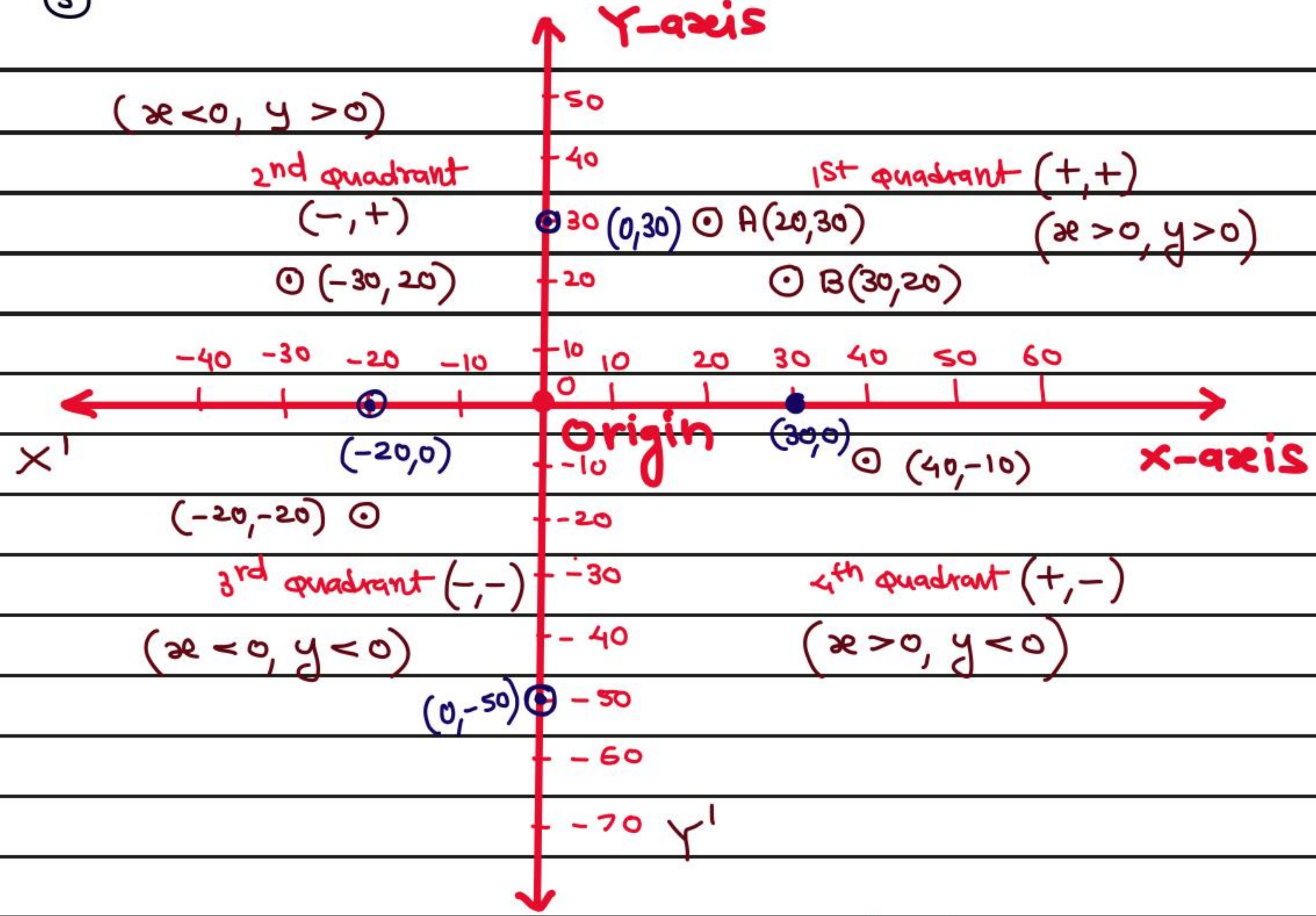

$A \equiv (20,30)$

For point A : 20 ⇒ x co-ordinate

30 ⇒ y co-ordinate

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points	Location	Equation / inequalities
$(+,+)$	1st quadrant	$x>0, y>0$
$(-,+)$	2nd quadrant	$x<0, y>0$
$(-,-)$	3rd quadrant	$x<0, y<0$
$(+,-)$	4th quadrant	$x>0, y<0$
$(\pm,0)$	X-axis	$y=0$
$(0,\pm)$	Y-axis	$x=0$
$(0,0)$	origin	$x,y=0$

- Equation of X-axis is $y = 0$
- Equation of Y-axis is $x = 0$
- $(0,0)$ represents origin = point of intersection of X, Y-axis
- If x-co-ordinate of a point is 0, then that point is on Y-axis
- If y-coordinate of a point is 0, then that point is on X-axis

⑤ Find points satisfying the linear equation $2x + 3y = 300$

$\Longrightarrow$ If I put $x = 150, y = 0$ then $2x + 3y = 300$ is satisfied $\therefore$ $(150, 0)$ is one of the point satisfying the eq^n $2x + 3y = 300$

other points : $(0,100)$, $\left(10, \frac{280}{3}\right)$, $\left(20, \frac{260}{3}\right)$, $(15,90)$, $(300,-100)$, $(60,60)$, $(75,50)$, $\left(40, \frac{220}{3}\right)$, $\left(-19, \frac{338}{3}\right)$ ---------

such infinite points can satisfy this linear equation.

⑥ Find points satisfying the linear equation $x + y = 50$

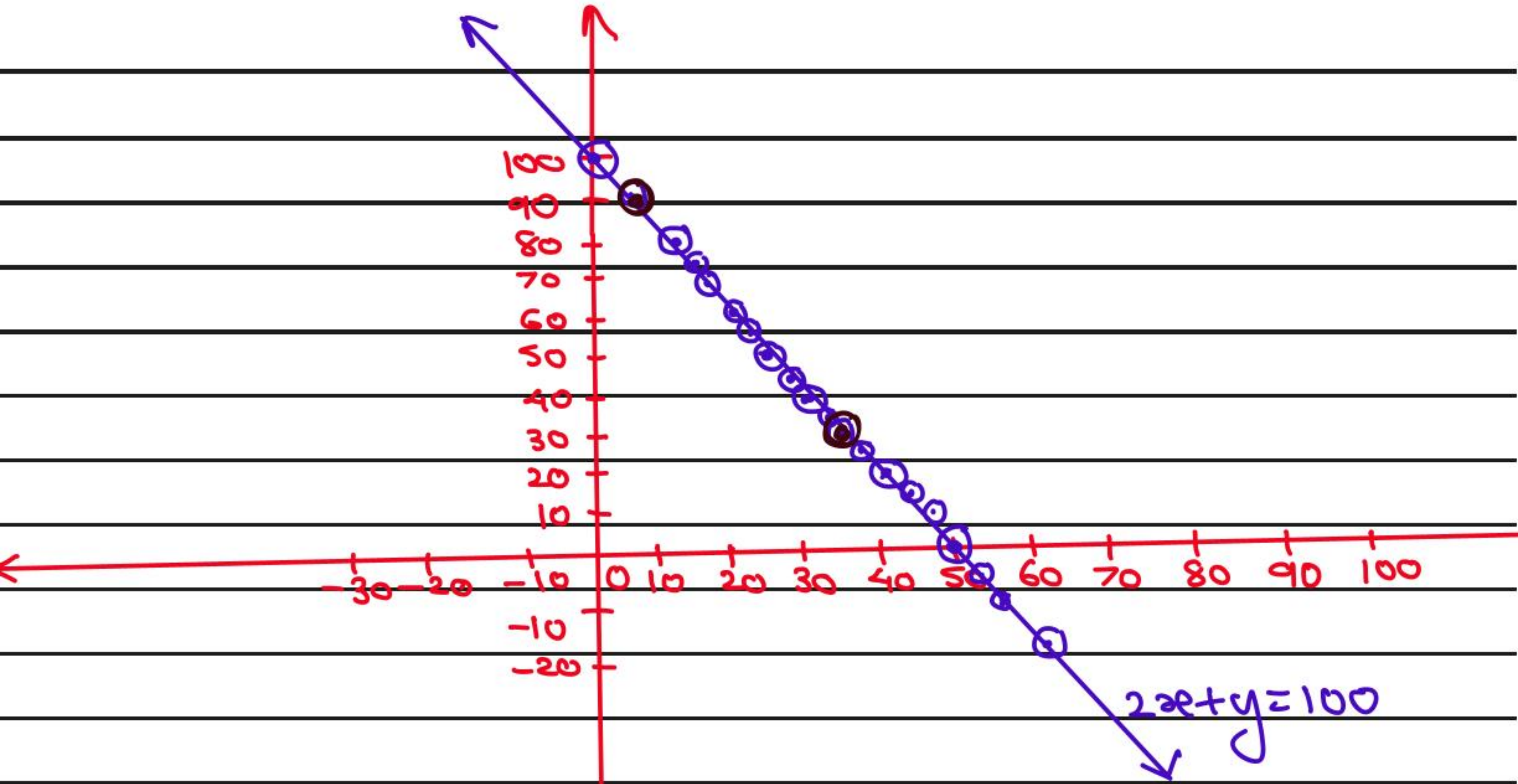
$\Longrightarrow$ $(0,50)$, $(50,0)$, $(30,20)$, $(20,30)$, $(10,40)$, $(25,25)$, $(60,-10)$, $(-20,70)$, $(-100,150)$, $(250,-200)$, $(1.50, 48.50)$, $(2.85, 47.15)$ $(1,49)$ $(15,35)$, $(45,5)$ ---------

such infinite points can satisfy this equation.

⑦ Find points satisfying the linear equation $2x + y = 100$ and plot those points on graph paper?

$\Longrightarrow$ points satisfying $2x + y = 100$ are :

$(50,0)$, $(0,100)$, $(30,40)$ $(40,20)$, $(60,-20)$, $(25,50)$, $(10,80)$, $(15,70)$, $(35,30)$ ---------



$\therefore$ Line is a set/collection of infinite points satisfying the given linear equation

- Graphical presentation of a linear equation is known as Line.

(8) How to draw a line on a Graph paper if equation of the line is given?

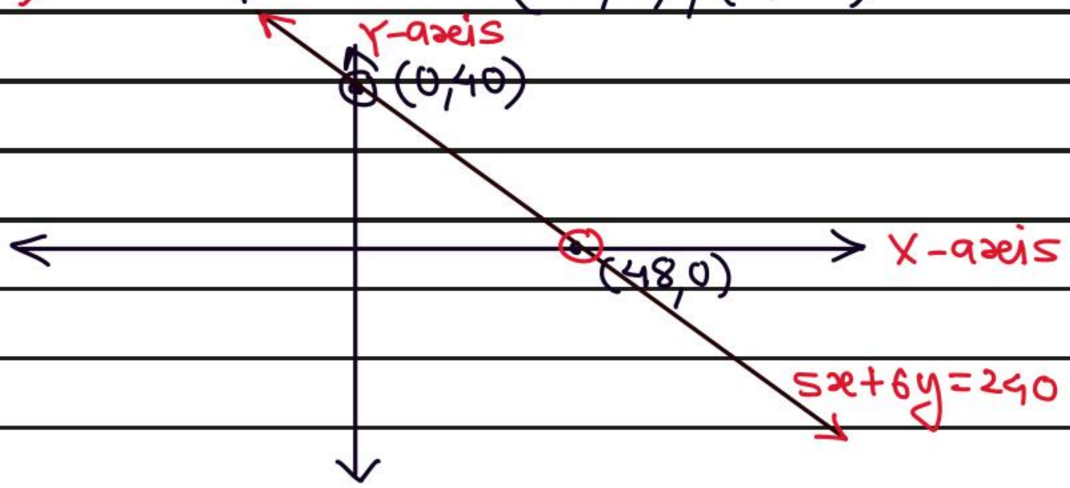
$\Longrightarrow$ (1) First find atleast 2 points satisfying the given linear equation.

(2) plot those points on Graph paper

(3) Draw a straight line passing through those points

⑨ Draw the line $5x + 6y = 240$ on graph paper.

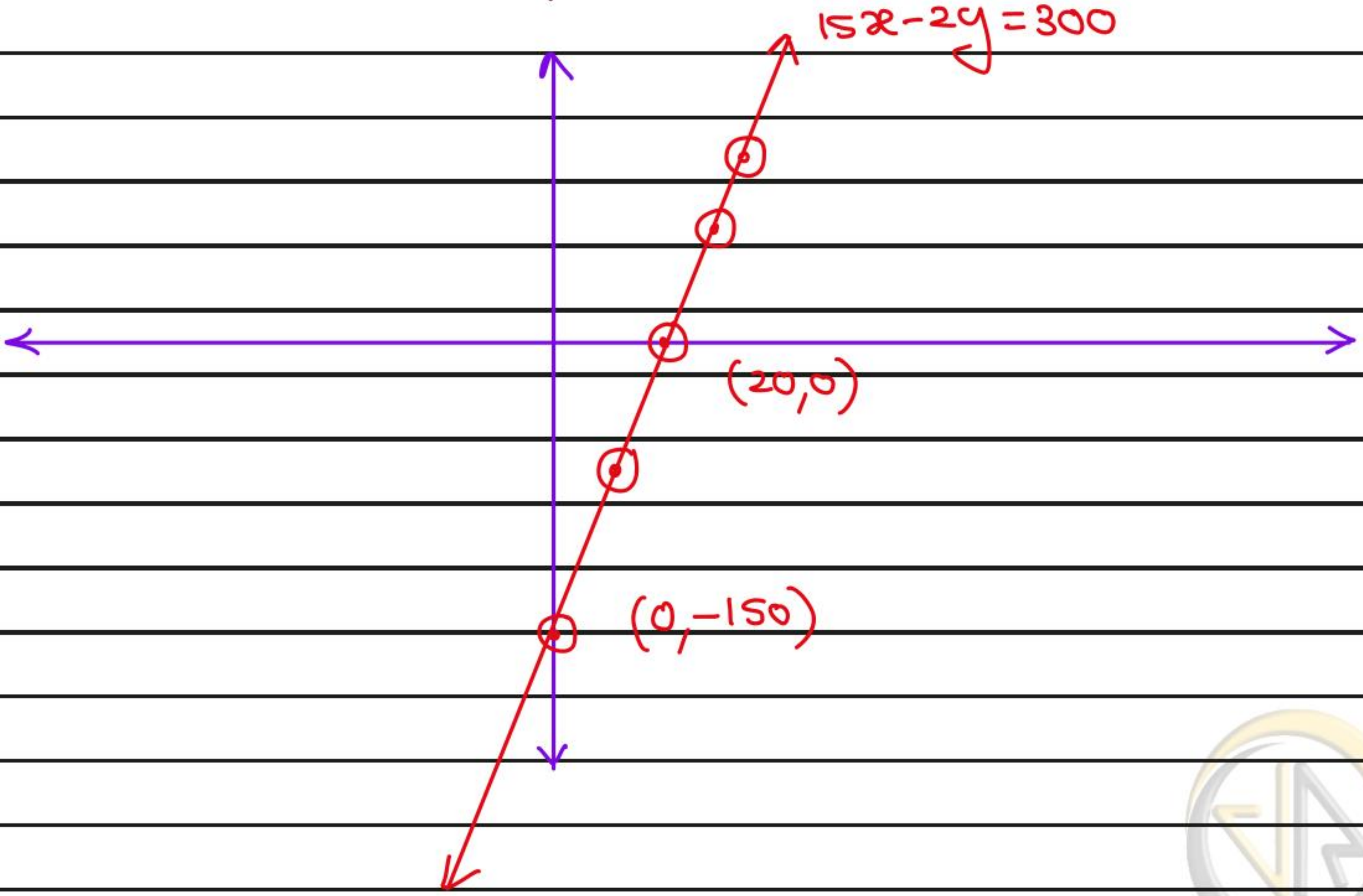
$\Longrightarrow$ points : $(48,0), (0,40)$



⑩ Draw the line $15x - 2y = 300$ on graph paper.

$\Longrightarrow$

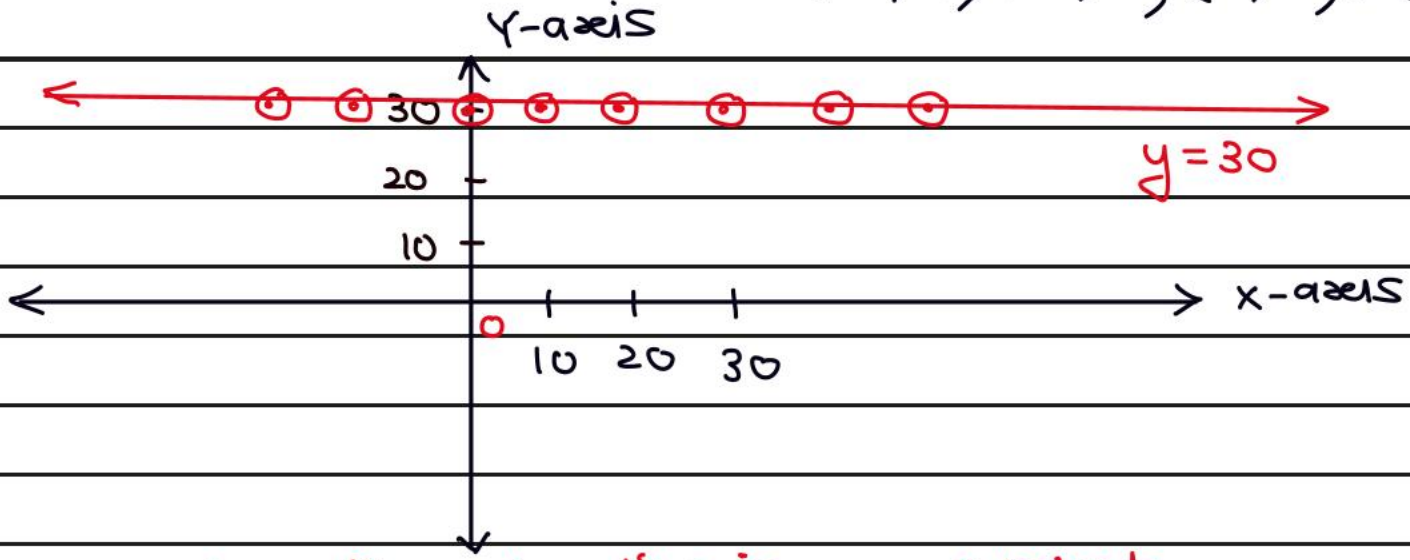
points satisfying the eqns $15x - 2y = 300$ are $(0,-150), (20,0)$



(11) Draw the line $y=30$ on graph paper.

$\Longrightarrow$ points satisfying the eq^n

$0x + y = 30 \Longrightarrow (0,30), (10,30), (20,30), (40,30), (-10,30), (-50,30), (30,30), (100,30)$

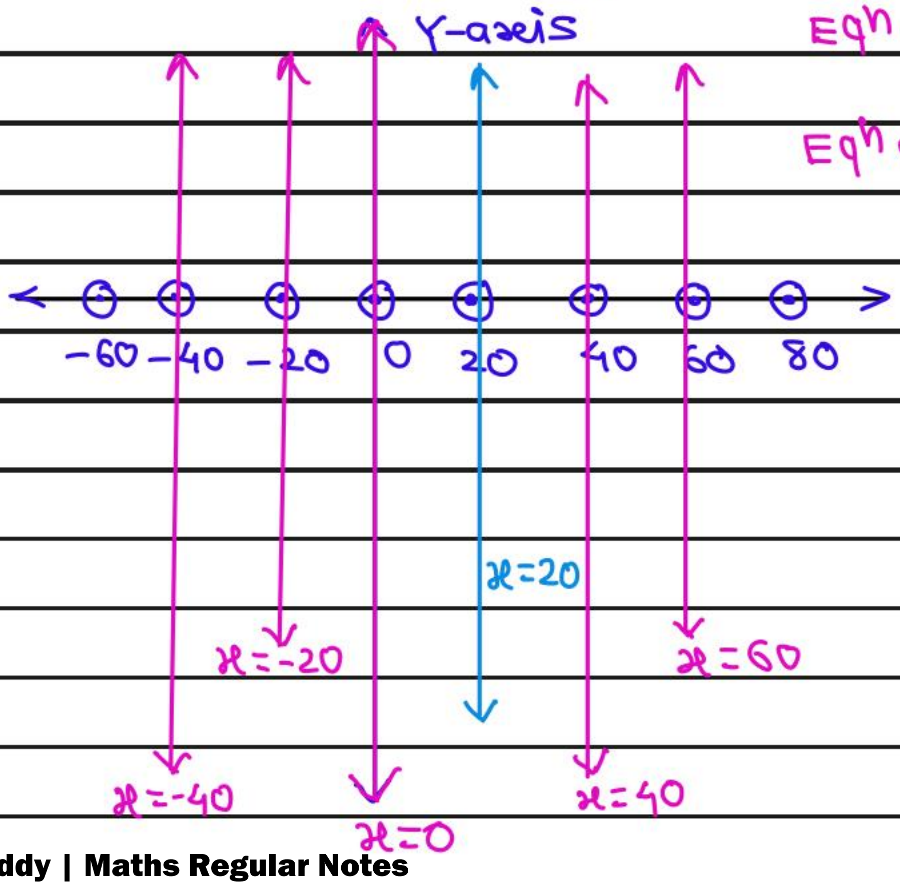


- If Eq^n of the line is y = constant then that line is $\parallel$ to X-axis
- If Eq^n of the line is x = constant then that line is $\parallel$ to Y-axis

(12)

Eq^n of X-axis : $y=0$

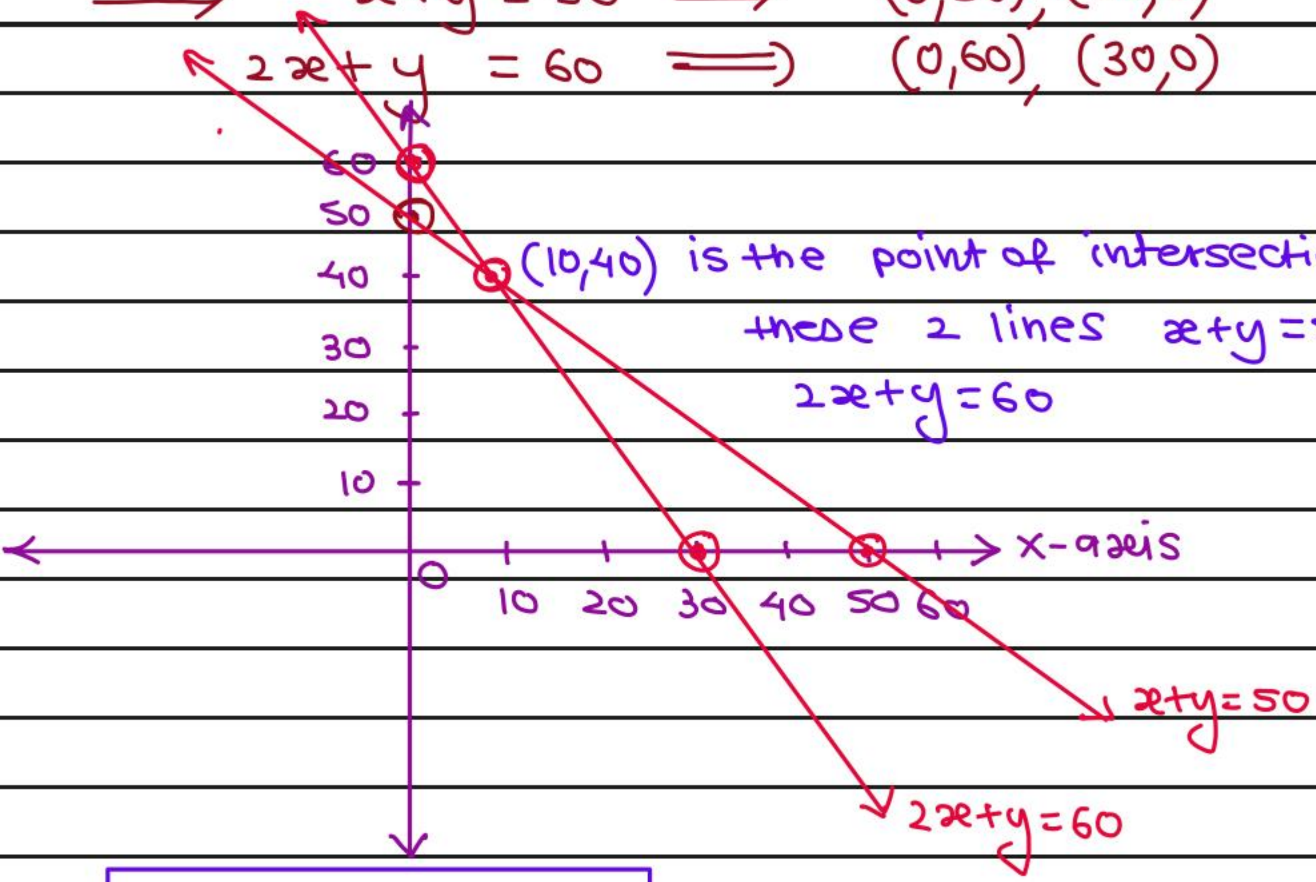
Eq^n of Y-axis : $x=0$



(13) Draw the lines $(x+y=50)$ & $(2x+y=60)$ on graph paper and Find point of intersection of these 2 lines.

$\Longrightarrow$ $x+y=50 \Longrightarrow (0,50),(50,0)$

$2x+y=60 \Longrightarrow (0,60),(30,0)$



(10,40) is the point of intersection of these 2 lines $x+y=50$ & $2x+y=60$

$$\begin{array}{r} x+\cancel{y}=50 \\ 2x+\cancel{y}=60 \\ - \quad - \quad - \\ \hline -x=-10 \\ x=10 \\ \therefore y=40 \end{array}$$

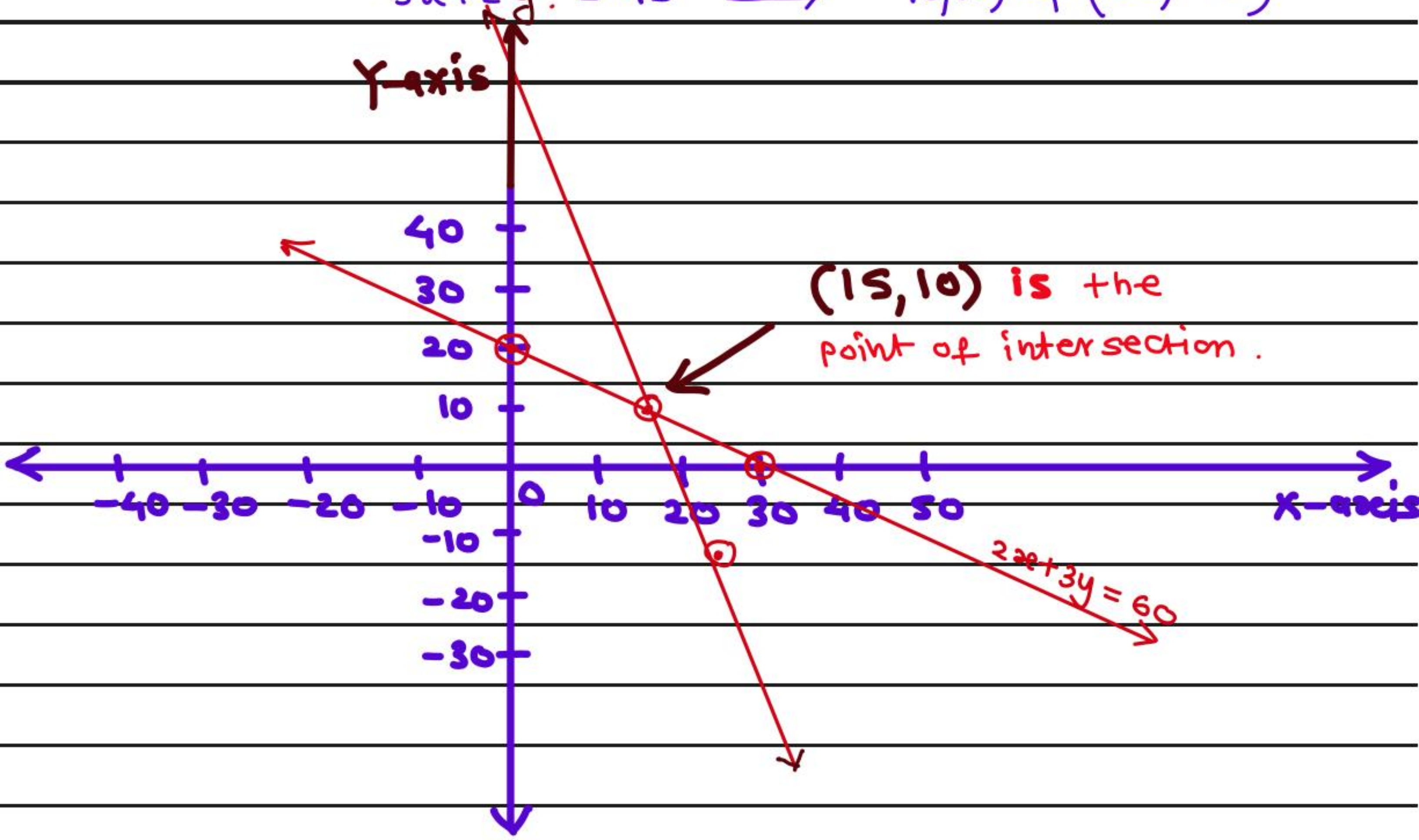
(10,40) is the point of intersection of Lines $x+y=50$ & $2x+y=60$

- on solving 2 linear eqns simultaneously if we get $x=p$, $y=q$ then (p,q) is the point of intersection of 2 lines.

(14) Draw the lines $2x+3y=60$ and and $5x+2y=95$ on graph paper and Find point of intersection.

$\Longrightarrow$ $2x+3y=60 \Longrightarrow (0,20)$ & $(30,0)$

$5x+2y=95 \Longrightarrow (15,10)$ & $(25,-15)$



To get the point of intersection, Let's solve 2 linear equations simultaneously

$$2x+3y=60 \text{ \& } 5x+2y=95$$

$$4x+\cancel{6y}=120$$
$$\underline{-15x\pm\cancel{6y}=-285}$$
$$-11x=-165$$
$$\boxed{x=15}$$

Let's put $x=15$ in one of the equation,

$$2x+3y=60$$
$$2(15)+3y=60$$
$$3y=30$$
$$\boxed{y=10}$$

$\therefore$ (15,10) is the point of intersection of lines $2x+3y=60$ & $5x+2y=95$

(15) Find point of intersection of $3x+5y=90$ & $2x+3y=60$

$\Longrightarrow$

$$\cancel{6x} + 10y = 180$$
$$\cancel{6x} + 9y = 180$$
$$- \quad - \quad -$$
$$y = 0$$

$$3x + 5(0) = 90$$
$$x = 30$$

$\therefore$ point of intersection $\equiv (30,0)$

(16) Find point of intersection of lines $3x+5y=100$ & $5x+3y=150$

$\Longrightarrow$

$$\cancel{15x} + 25y = 500$$
$$\cancel{15x} + 9y = 450$$
$$- \quad \pm \quad -$$
$$16y = 50$$
$$\boxed{y = 3.125}$$

$$3x + 5(3.125) = 100$$
$$\boxed{x = 28.125}$$

$\therefore$ $(28.125, 3.125)$ is the point of intersection

(17) point of intersection of $7x-3y=20$ & $5x+13y=200$ lie in _____ quadrant.

$\Longrightarrow$

$$\cancel{35x} - 15y = 100$$
$$\cancel{35x} + 91y = 1400$$
$$- \quad - \quad -$$
$$-106y = -1300$$
$$\boxed{y = 12.2642}$$

$$7x - 3(12.2642) = 20$$
$$\boxed{x = 8.1132}$$

$\therefore$ $(8.1132, 12.2642)$ is the point of intersection

$\therefore$ The point of intersection is in 1st quadrant.

(18) point of intersection of lines $5x+2y=90$

$10x+9y=180$ lie in ______ quadrant

(1) 1^{st} (2) 2^{nd} (3) 3^{rd} ~~(4)~~ None of these

$\Longrightarrow$

$$\cancel{10x} + 4y = 180$$
$$\cancel{10x} + 9y = 180$$
$$- \quad - \quad -$$
$$-5y = 0$$
$$y = 0$$
$$\therefore \quad 5x+2y=90$$
$$5x+2(0)=90$$
$$x=18$$

$(18,0)$ is the point of intersection

(19) Find point of intersection of

$8x-y=90$ & $3x-7y=190$

$\Longrightarrow$

$$56x - \cancel{7y} = 630$$
$$3x - \cancel{7y} = 190$$
$$- \quad + \quad -$$
$$53x = 440$$
$$\boxed{x = 8.3019}$$
$$8(8.3019)-90=y$$
$$\boxed{\therefore \quad y=-23.5848}$$

$\therefore$ point of intersection $\equiv (8.3019, -23.5848)$

(20) The point $(8k,-19)$ lie on the line $5x-13y=88$. Find value of k.

$\Longrightarrow$

$$5x-13y=88$$
$$5(8k)-13(-19)=88$$
$$40k+247=38$$
$$\boxed{k=-3.975}$$

(21) The point $(-k/3, 35)$ lie on the line $10x - 55y = 230$. Find value of k.

$\Longrightarrow$ $10x - 55y = 230$

$10(-k/3) - 55(35) = 230$

$\frac{-10k}{3} = 230 + 1925$

$\frac{-10k}{3} = 2155$

$k = -646.50$

(22) Find point of intersection of lines $2x+3y = 800$ and $8x + 12y = 1000$

$\Longrightarrow$ $8x + 12y = 3200$
$8x + 12y = 1000$ } These 2 lines are parallel (||) to each other

$\therefore$ There is no solution (i.e. There is no point of intersection)

$$\boxed{\text{slope of the line } ax + by + c = 0 \text{ is } (-a/b)}$$

slope of the line $2x+3y = 800$ is $-2/3$

slope of the line $8x+12y = 1000$ is $-8/12 = -2/3$

As slope of 1^{st} line = slope of 2^{nd} line,

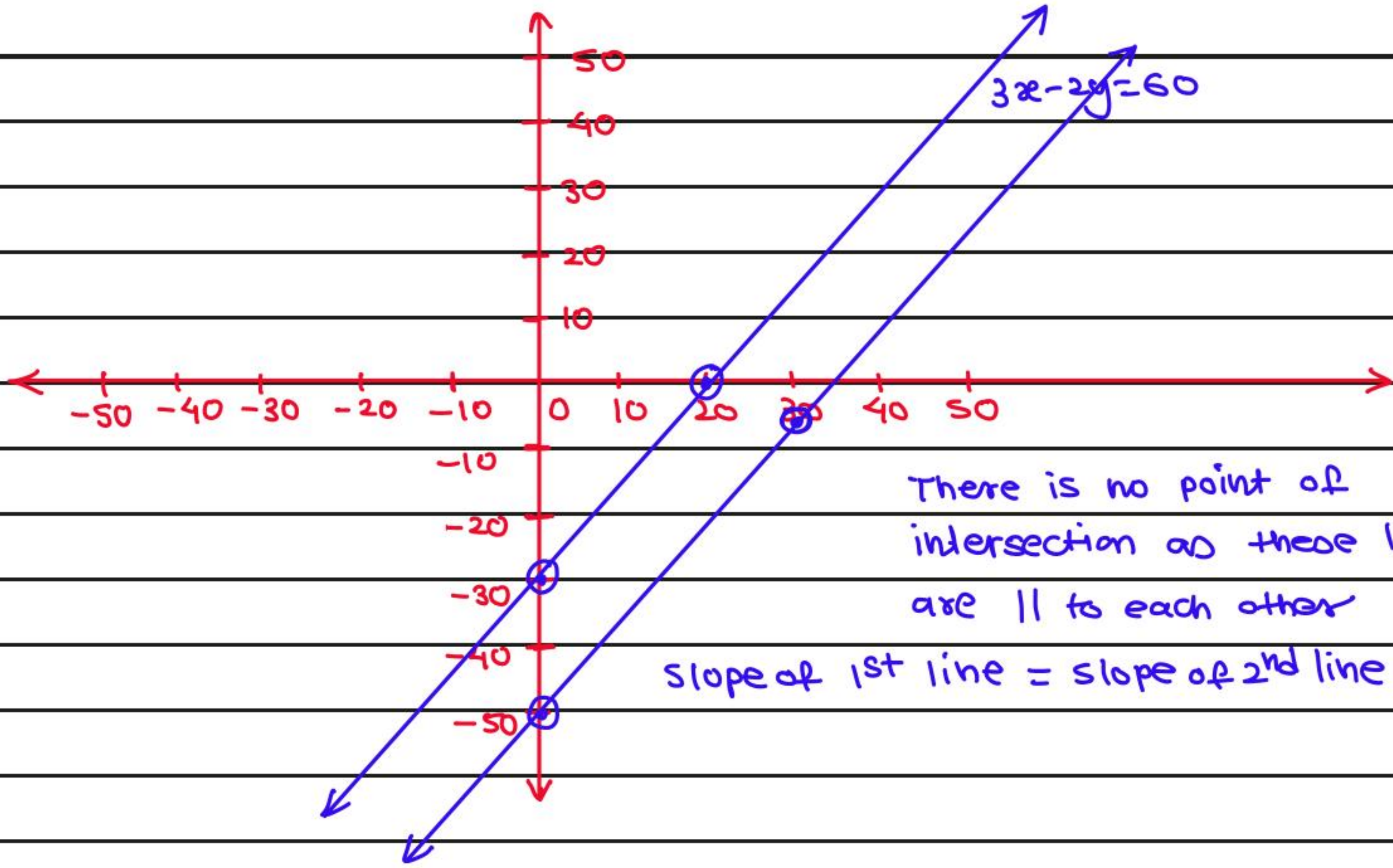
Lines are || to each other.

If m_1 is slope of one line & m_2 is slope of other line then lines are said to be || to each other If $(m_1 = m_2)$

(23) Draw the lines $3x-2y=60$ & $9x-6y=300$ on graph paper. Find point of intersection.

$\Longrightarrow$ $3x-2y=60 \Longrightarrow (20,0), (0,-30)$

$9x-6y=300 \Longrightarrow (0,-50), (30,-5)$



There is no point of intersection as these lines are || to each other

slope of 1st line = slope of 2nd line = $3/2$

(24) The lines $5x+11y=22$ and $8kx-55y=-980$ are || to each other. Find value of k.

$\Longrightarrow$ As these lines are || to each other

slope of 1st line = slope of 2nd line

$$-5/11 = 8k/55$$

$$k = -\frac{5}{\cancel{11}} \times \frac{\cancel{55}^{5}}{8} = -25/8$$

$$k = -3.125$$

(25) The lines $5x+13y=80$ & $8mx-22y=810$ and || to each other. Find m.

$\Longrightarrow$ slope of 1st line = slope of 2nd line

$$-5/13 = 8m/22$$

$$8m = -\frac{5}{13} \times 22$$

$$\boxed{m = -1.0577}$$

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Eqn of the line	Slope of the line
$ax + by - c = 0$	$-a/b$
$3x + 5y + 30 = 0$	$-3/5$
$3x + 5y - 1000 = 0$	$-3/5$
$5x - 13y = 88$	$5/13$
$8kx - 33py = 8k - p$	$8k/33p$
$29x - 33y = 5x - 88$ $24x - 33y = -88$	$24/33 = 8/11$
$13x - 2y = 88x - 130y - y + 2x - 8p + 63$ i.e. $-77x + 129y = -8p + 63$	$77/129$
$31x - 2y = 8kx - 55y + 11$ ie. $(31-8k)x + 53y = 11$	$\frac{-(31-8k)}{53} = \frac{(8k-31)}{53}$
$x = 35$ $x + 0y - 35 = 0$	$-1/0$ = Not defined
$2x = 101$ $2x + 0y - 101 = 0$	$-2/0$ = Not defined
$5y = 33$ $0x + 5y = 33$	$-0/5 = 0$
$y = 33$	0
$x = 500$	Not defined
$px + qy + r = 0$	$-p/q$
$33x + py = r$	$-33/p$

(27) Find slope of the line $x = 155$ i.e. $x + 0y = 155$

slope = Not defined

(28) Find slope of the line $y = 30$ i.e. $0x + y = 30$

slope = zero

slope of x-axis and all the lines $\parallel$ to x-axis is : zero
slope of Y-axis & all the lines $\parallel$ to Y-axis is : not defined

A line	slope	Equation
X – axis	0	$y = 0$
Y – axis	Not defined	$x = 0$
$\parallel$ line to X-axis	0	$y = \text{constant}$
$\parallel$ line to Y-axis	Not defined	$x = \text{constant}$

(29) Standard format of a linear equation is,

$$ax + by + c = 0$$

$$by = -ax \pm c$$

dividing by 'b' on both sides

$$\frac{by}{b} = \frac{-ax \pm c}{b}$$

$$y = \left(\frac{-a}{b}\right)x + \text{constant}$$

$$y = mx + c$$

where m = slope of the line

(30) Find slope of the line $3x+5y=88$

$3x+5y-88=0$

comparing this with $ax+by+c=0$

$a=3,\ b=5$

$\therefore$ slope $= -a/b$

$= -3/5$

$3x+5y=88$

$5y = 88-3x$

dividing by '5' on both sides

$y = \left(-\frac{3}{5}\right)x + \frac{88}{5}$

comparing this with $y = mx+c$

$m = -3/5$

(31) Find any 2 points satisfying the Equation $7x-3y=100$

$\Rightarrow$

$(100, 200)\ (10, -10)$

(32) Find Eqn of the line passing through points $(100,200)$ & $(10,-10)$

$\Rightarrow$ (x_1, y_1) (x_2, y_2)

$$\left(\frac{y_2-y_1}{y-y_1}\right) = \left(\frac{x_2-x_1}{x-x_1}\right) \quad \text{-----} \quad \left[\text{Eqn of line passing through } (x_1, y_1) \text{ \& } (x_2-y_2)\right]$$

$$\left(\frac{-10-200}{y-200}\right) = \left(\frac{10-100}{x-100}\right)$$

$$\frac{-210}{y-200} = \frac{-90}{x-100}$$

$$-210x + 21000 = -90y + 18000$$

$$3000 = 210x - 90y$$

$210x - 90y = 3000$

$7x - 3y = 100$

(33) Find Equation of the line passing through points (x_1, y_1) & (x_2, y_2)

$\Longrightarrow$ Let $y = mx + c$ be the eqn of the line

passing through point (x_1, y_1) & (x_2, y_2)

$y = mx + c$ ---------- ①

As point (x_1, y_1) is on the line $y = mx + c$

$y_1 = mx_1 + c$ ---------- ②

similarly $y_2 = mx_2 + c$ ---------- ③

eqn ① − eqn ②

$y - y_1 = mx + \cancel{c} - mx_1 - \cancel{c}$

$y - y_1 = m(x - x_1)$

$\therefore \quad m = \left(\dfrac{y - y_1}{x - x_1}\right)$ ---- ④

eqn ③ − eqn ②

$y_2 - y_1 = mx_2 + \cancel{c} - mx_1 - \cancel{c}$

$y_2 - y_1 = m(x_2 - x_1)$

$\therefore \quad m = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)$ ---- ⑤

$\therefore$ From Eqns ④ & ⑤

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\therefore \left(\frac{y_2 - y_1}{y - y_1}\right) = \left(\frac{x_2 - x_1}{x - x_1}\right)$$ -----> This is eqn of the line passing through points (x_1, y_1) & (x_2, y_2)

(34) Find Eqn of the line passing through point (p, q), (m, n)

$$\Longrightarrow \left(\frac{n - q}{y - q}\right) = \left(\frac{m - p}{x - p}\right)$$

(35) Find slope of the line $y = -\frac{8}{5}x + 33$

$\Longrightarrow$

$y = -\frac{8}{5}x + 33$

comparing this with

$y = mx + c$

$m = -8/5 = $ slope of the line

$y = \frac{-8x}{5} + 33$

$y - 33 = -\frac{8x}{5}$

$5y - 165 = -8x$

$8x + 5y = 165$

$8x + 5y - 165 = 0$

$\Downarrow$

comparing this with

$ax + by + c = 0$

$a = 8,\ b = 5$

slope $= -a/b = -8/5$

(36) slope of the line $kx + 15y = 2x - 93$ is $-8/11$
Find k.

$\Longrightarrow$ $kx + 15y = 2x - 93$

$kx - 2x + 15y = -93$

$(k-2)x + 15y = -93$

slope $= \frac{-(k-2)}{15} = \frac{-8}{11}$

$\frac{k-2}{15} = \frac{8}{11}$

$11k - 22 = 120$

$11k = 142$

$\boxed{k = 12.909090}$

(37) slope of the line $19x - 33y + 2ky = 8x - 930$
is $(11/8)$ Find k.

$$\Rightarrow \quad 19x - 33y + 2ky - 8x + 930 = 0$$

$$11x + (2k-33)y + 930 = 0$$

$$\text{Slope} = \frac{-11}{2k-33} = \frac{11}{8}$$

$$22k - 363 = -88$$

$$22k = 275$$

$$k = \left(\frac{275}{22}\right)$$

$$k = 12.50 = 12\tfrac{1}{2} = \left(\frac{25}{2}\right)$$

(38) Find Eq^n of the line passing through points $(8,-12), (18,33)$

$\Rightarrow$ Eq^n of the line passing through points (x_1, y_1) & (x_2, y_2) is $\left(\dfrac{y_2-y_1}{y-y_1}\right) = \left(\dfrac{x_2-x_1}{x-x_1}\right)$

$$\frac{33+12}{y+12} = \frac{18-8}{x-8}$$

$$45(x-8) = 10(y+12)$$

$$45x - 360 = 10y + 120$$

$$45x - 10y = 480$$

$$\boxed{9x - 2y = 96}$$

(39) Find Eq^n of line passing through points $(-30,-20), (-1.50, 80)$

$$\Rightarrow \quad \frac{100}{y+20} = \frac{28.50}{x+30}$$

$$100x + 3000 = 28.50y + 570$$

$$100x - 28.50y = -2430$$

$$1000x - 285y = -24{,}300$$

$$\boxed{200x - 57y = -4860}$$

(40) Find Eqⁿ of the line passing through points $(2,-5)$, $(-11,20)$. Also Find slope of that line.

$$\frac{25}{y+5} = \frac{-13}{x-2}$$

$$25x - 50 = -13y - 65$$

$$25x + 13y = -15$$

$$25x + 13y + 15 = 0$$

$\Downarrow$

slope of the line $= -25/13$

slope of the line passing through points (x_1, y_1) & (x_2, y_2)

$$= \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\text{slope} = \frac{20-(-5)}{-11-2} = \frac{25}{-13} = -25/13$$

Eqⁿ of line:

$$25x + 13y = -15$$

(41) Find slope, Eqⁿ of the line passing through points $(20, 28)$, $(30, 85)$

$\Longrightarrow$ slope of the line $= \dfrac{85-28}{30-20} = \left(\dfrac{57}{10}\right)$

Eqⁿ of the line

$$57x - 10y = 57(20) - 10(28)$$

$$\boxed{57x - 10y = 860}$$

OR $\quad -57x + 10y = -860$

(42) Find slope, Eqⁿ of the line passing through points $(1.50, 18.50)$, $(-27, 35)$

$\Longrightarrow$

$$\text{slope} = \left(\frac{35 - 18.50}{-27 - 1.50}\right) = \frac{16.50}{-28.50} = \frac{-165}{285}$$

$$= -33/57 = -11/19$$

Eqⁿ of Line $\Longrightarrow$ $11x + 19y = 368$

(43) Find slope of the line passing through points (a,b) & (c,d)

(a) $\left(\frac{d-b}{c-a}\right)$ (b) $\left(\frac{b-d}{a-c}\right)$ ~~(c)~~ Both (d) None

(44) Slope of the line passing through $(2k,19)$ & $(50,-8)$ is $-16/3$ Find k.

$$\Rightarrow \quad \text{slope} = \frac{-8-19}{50-2k} = \frac{-16}{3}$$

$$\frac{-27}{50-2k} = \frac{-16}{3}$$

$$-81 = -800+32k$$

$$719 = 32k$$

$$k = \left(719/32\right) = 22.46875$$

(45) The line $8x-3y=20$ & $7kx+55y=250$ have no solution. Find k.

$\Rightarrow$ As these 2 line has no solution means these 2 lines do not intersect with each other.

$\therefore$ They are $\parallel$ to each other

$\therefore$ slope of 1st line = slope of 2nd line

$$\frac{8}{3} = \frac{-7k}{55}$$

$$-21k = 440$$

$$k = \left(\frac{-440}{21}\right) = -20.9524$$

(46) The lines $5x + 11y = 29$ & $kx + 33y = 810$ have unique solution then

(a) $k = 15$ (b) $k \neq 15$ (c) $k = 0$ (d) wrong question

They have point of intersection means,

slope of 1st line $\neq$ slope of 2nd line

$$-5/11 \neq -k/33$$

$$\frac{5}{11} \neq \frac{k}{33}$$

$$k \neq 15$$

(47) If m_1 is slope one line & m_2 is slope of other line then Lines are said to be

|| to each other when $m_1 = m_2$

oblique when $m_1 \neq m_2$

$\perp$ to each other when $m_1 \times m_2 = -1$

(48) $3x - 19y = 50$ & $2kx + 51y = 200$ are $\perp$ to each other. Find value of k.

$\Longrightarrow$ slope of 1st line $\times$ slope of 2nd line $= -1$

$$\frac{3}{19} \times -\frac{2k}{51} = -1$$

$$\frac{-6k}{969} = -1$$

$$-6k = -969$$

$$k = \frac{969}{6} = \frac{323}{2} = 161.50$$

(49)

Slope of the line	slope of its ∥ line	Slope of its ⊥ line
$3/5$	$3/5$	$-5/3$
$-8/9$	$-8/9$	$9/8$
8	8	$-1/8$
-11	-11	$1/11$
$33/8$	$33/8$	$-8/33$
$-p/q$	$-p/q$	q/p
$(p-q)/r$	$(p-q)/r$	$-r/(p-q) = \left(r/(q-p)\right)$
0	0	Not defined
Not defined	Not defined	0
$3/91$	$3/91$	$-91/3$

(50) The lines $18x - my = 20$ & $51x - 28y = 290$ are ⊥ to each other. Find value of m.

$\Longrightarrow$ Slope of 1st line x slope of 2nd line = −1

$$\frac{18}{m} \times \frac{51}{28} = -1$$

$$\frac{918}{28m} = -1$$

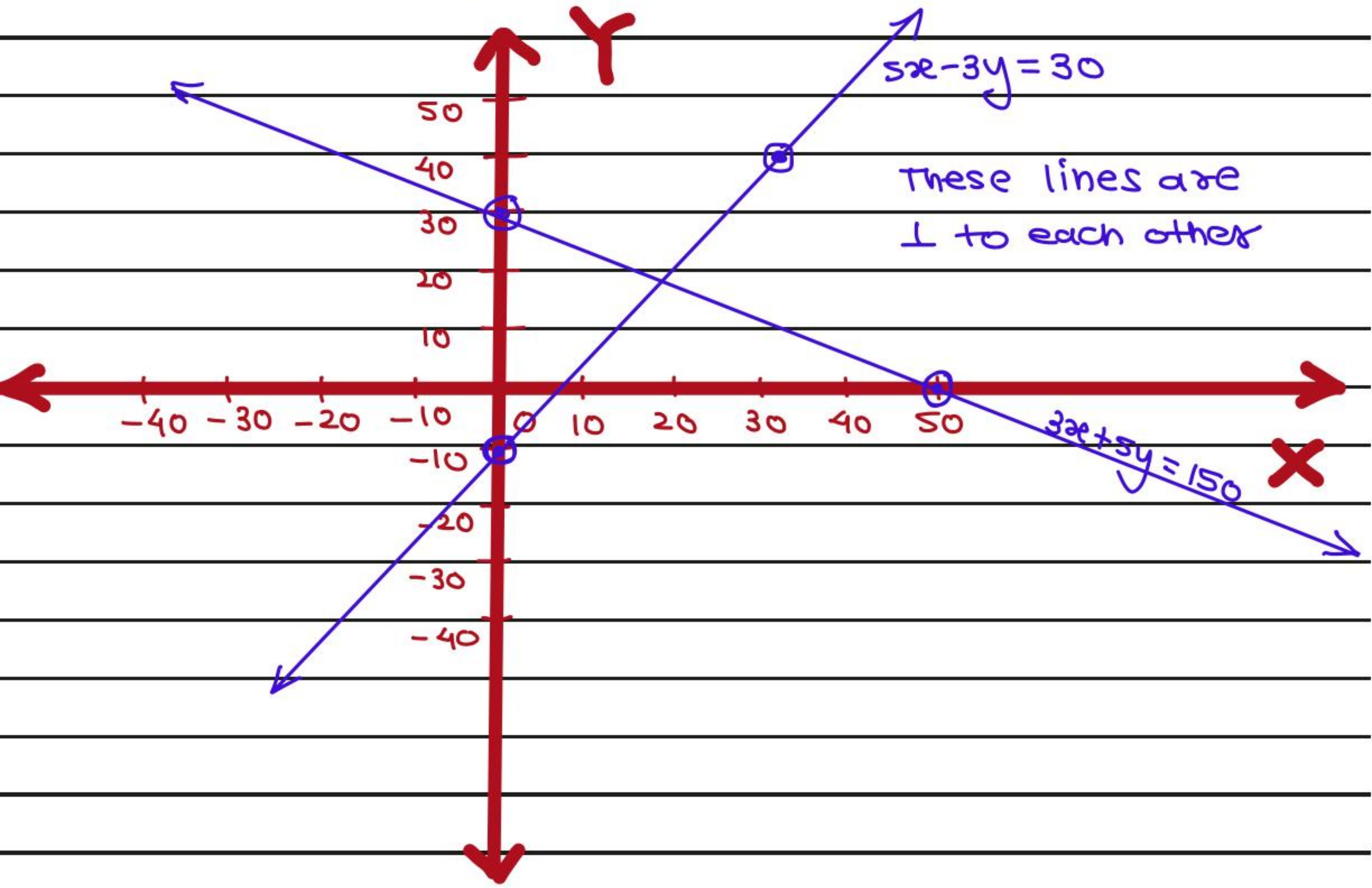
$$918 = -28m$$

$$m = -\frac{918}{28} = -32.7857$$

(51) Draw the line $3x+5y=150$ & $5x-3y=30$ on graph paper

$\Longrightarrow$ $3x+5y=150 \Longrightarrow (50,0),(0,30)$

$5x-3y=30 \Longrightarrow (0,-10),(30,40)$



These lines are $\perp$ to each other

(52) The lines $8x-3ky+21y=33$ and $15x-28y=233$ are $\perp$ to each other. Find k.

$\Longrightarrow$ $8x-3ky+21y=33$

$8x+(21-3k)y=33$ & $15x-28y=233$

are $\perp$ to each other

$$\frac{-8}{21-3k} \times \frac{15}{28} = -1$$

$$\frac{-120}{588-84k} = -1$$

$$\frac{120}{588-84k} = 1$$

$$120 = 588 - 84k$$

$$84k = 468$$

$$k = \left(\frac{468}{84}\right) = \left(\frac{117}{21}\right) = \left(\frac{39}{7}\right)$$

$$k = 5.571428$$

(53) Find Eq^n of line passing through point (8,20) having slope of (−0.60)

$\Rightarrow$

Slope $= \left(-0.60/1\right)$

Eq^n of the line

$$0.60x + y = 0.60(8) + 20$$

$$0.60x + y = 24.80$$

$$6x + 10y = 248$$

$$\boxed{3x + 5y = 124}$$

Slope $= -0.60$

$= -3/5$

Eq^n of the line

$$3x + 5y = 3(8) + 5(20)$$

$$\boxed{3x + 5y = 124}$$

(54) Find Eq^n of the line passing through point (8,10) having slope of 0.70

$\Rightarrow$ Slope $= 0.70 = \left(7/10\right)$

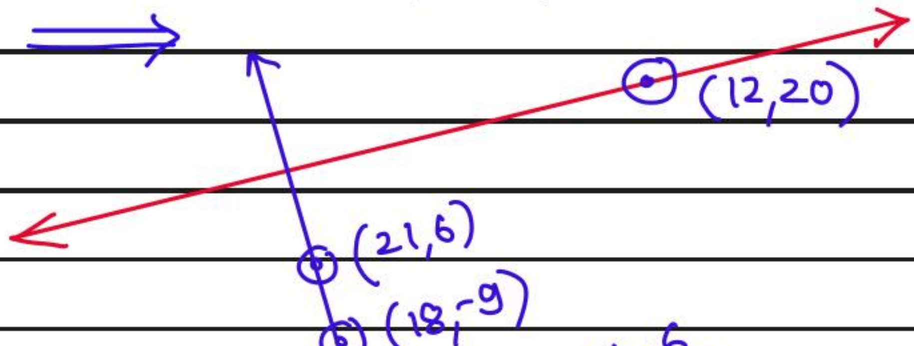
Eq^n of the line : $7x - 10y = -44$

OR

$$-7x + 10y = 44$$

(55) Find Eq^n of line passing through point (12, 20) and $\perp$ to line joining (21, 6), (18, −9)

$\Longrightarrow$



$$\text{slope} = \frac{-9-6}{18-21} = \frac{-15}{-3} = 5$$

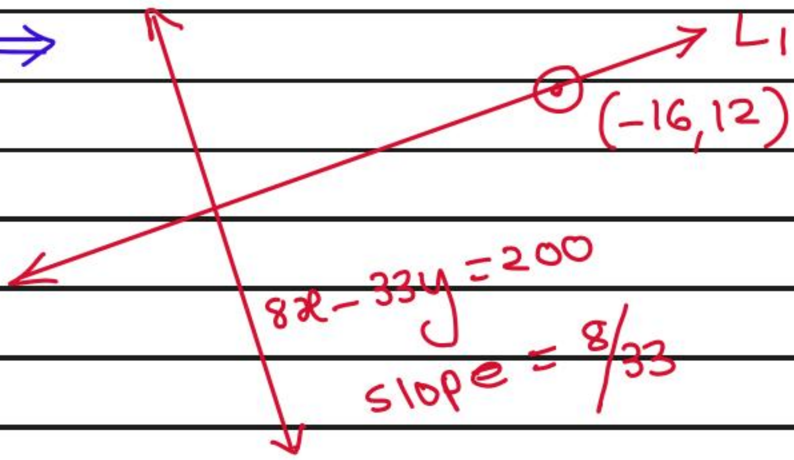
slope of $L_1 = -1/5$

<u>Eq^n of L1</u>

$x + 5y = 112$

(56) Find Eq^n of line passing through point (−16, 12) and $\perp$ to $8x - 33y = 200$

$\Longrightarrow$



$\text{slope} = 8/33$

slope of $L_1 = -33/8$

<u>Eq^n of L1</u>

$33x + 8y = -432$

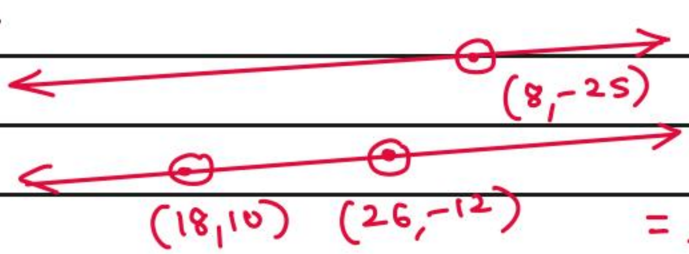
$33x + 8y + 432 = 0$

(57) Find eq^n of the line passing through point (8, −25) and $\parallel$ to line joining (18, 10), (26, −12)

slope of $L_1 = -11/4$

$\Longrightarrow$

Eq^n: $\boxed{11x + 4y = -12}$



slope

$$= \frac{-12-10}{26-18} = \frac{-22}{8} = -11/4$$

(58) Find Eqⁿ of line having slope of 8/5 and passing through point (20,16)

$\Longrightarrow$

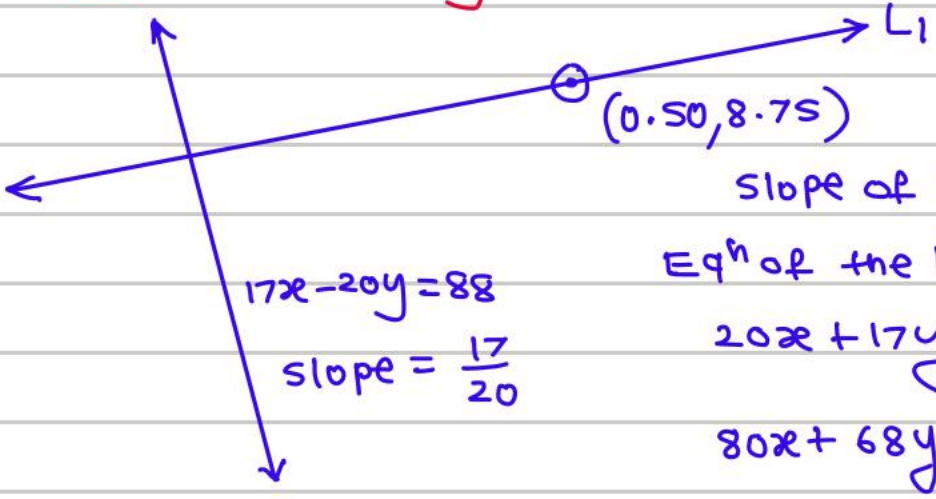
$$8x - 5y = 8(20) - 5(16)$$

$$8x - 5y = 80$$

$$8x - 5y - 80 = 0$$

(59) Find Eqⁿ of the line passing through (0.50, 8.75) and $\perp$ to $17x - 20y = 88$

$\Longrightarrow$



Slope of $L_1 = -\frac{20}{17}$

Eqⁿ of the line

$$20x + 17y = 158.75$$

$$80x + 68y = 635$$

(60) If slope of line is zero then that line can be

(a) X-axis (b) || to x-axis (c) $\perp$ to Y-axis

~~(d)~~ All of these

(61) If slope of a line is Not defined then that line can be

(a) Y-axis (b) || to Y-axis (c) $\perp$ to x-axis

~~(d)~~ All of these

(62) The line $x = 25/2$ is ______

(a) || to Y-axis (b) $\perp$ to x-axis

~~(c)~~ Both (d) None

(63) Find Eq^n of line passing through points $(8,5), (9,5)$

$\Longrightarrow \quad y=5$

(64) Find Eq^n of line passing through points $(6,0), (19,0)$

$\Longrightarrow \quad y=0$

(65) Find Eq^n of line passing through point $(0,18), (18,0)$

$\Longrightarrow \quad x+y=18$

(66) Find Eq^n of line passing through points $(0,19), (5,19)$

$\Longrightarrow \quad y=19$

(67) Slope of the line passing through points $\left(\frac{8}{3}, \frac{7}{5}\right), \left(\frac{2k}{7}, \frac{19}{3}\right)$ is $5/11$. Find k.

$$\Longrightarrow \quad \frac{\frac{19}{3}-\frac{7}{5}}{\frac{2k}{7}-\frac{8}{3}} = \frac{5}{11}$$

$$\frac{\frac{74}{15}}{\frac{6k-56}{21}} = \frac{5}{11}$$

$$\frac{74}{15} \times \frac{21}{6k-56} = \frac{5}{11}$$

$$\frac{74}{15} \times 21 \times \frac{11}{5} = 6k-56$$

$$\boxed{k = 47.32}$$

(68) The lines $3kx - 22y = 80$ & $90x - 47y = 285$ are $\perp$ to each other. Find k.

$\Longrightarrow$ Slope of 1^{st} line $\times$ slope of 2^{nd} line $= -1$

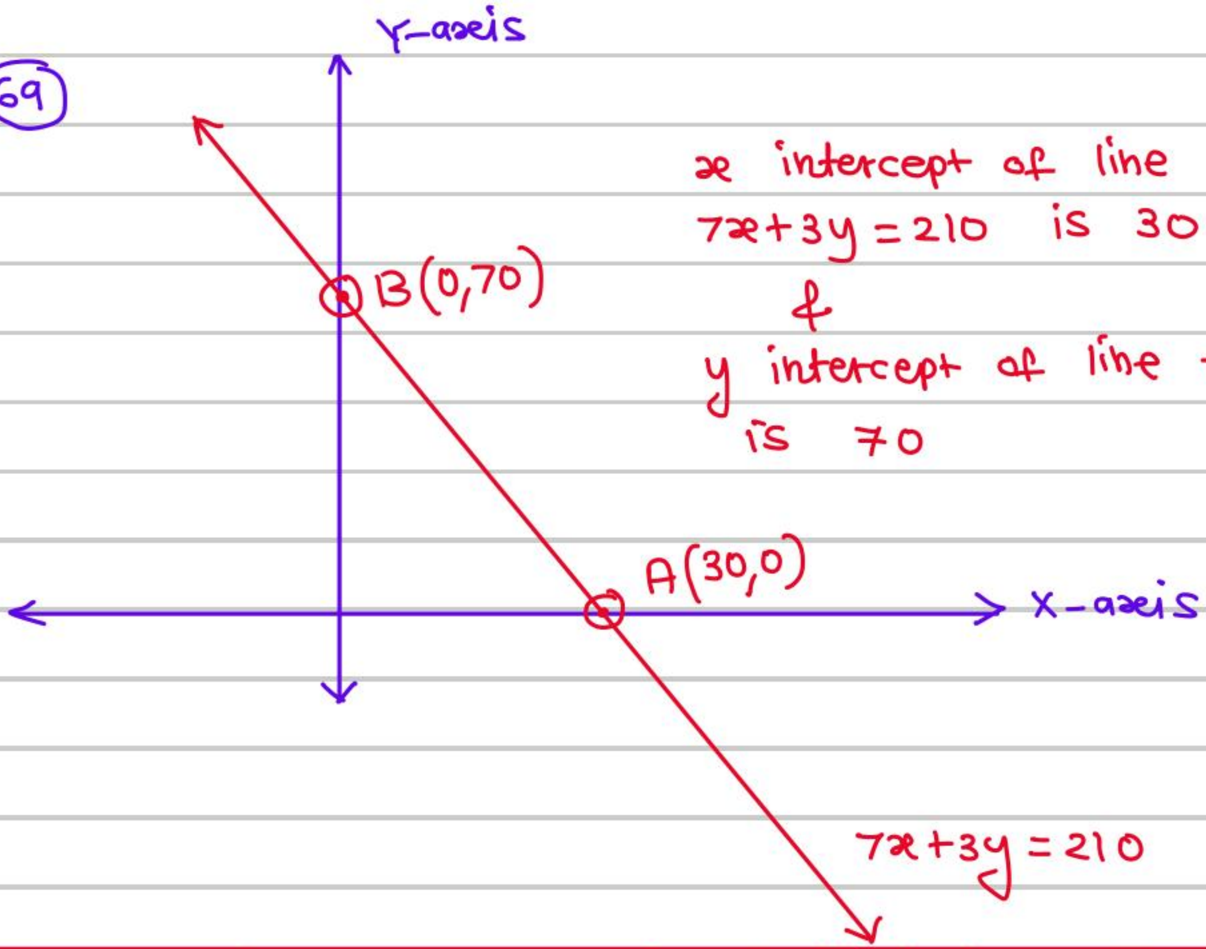
$$\frac{3k}{22} \times \frac{90}{47} = -1$$

$$\frac{3k}{22} = -\frac{47}{90}$$

$$k = -\frac{47}{90} \times \frac{22}{3}$$

$$k = -3.8296$$

(69)



x intercept of line $7x+3y=210$ is 30

&

y intercept of line $7x+3y=210$ is 70

$$\boxed{\text{If x intercept of a line is 'm' \& y intercept is k then that line passes through (m,0) \& (0,k)}}$$

(70) Find Eq^n of line having x, y intercepts as a, b respectively

$\Longrightarrow$ this Line passes through $(a,0)$ & $(0,b)$

$\therefore$ Eq^n of line passing through points $(a,0)$ & $(0,b)$ is

$$\frac{b-0}{y-0} = \frac{0-a}{x-a}$$

$$\frac{b}{y} = \frac{-a}{x-a}$$

$$b(x-a) = -ay$$

$$bx - ab = -ay$$

$$bx + ay = ab$$

dividing by 'ab' on both sides

$$\frac{bx}{ab} + \frac{ay}{ab} = \frac{ab}{ab}$$

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1} \Longleftarrow \text{Intercept form of Eq}^n \text{ of the Line}$$

where $a = x$ intercept
$b = y$ intercept

(71) Find Eq^n of line passing through points $(30,0)$, $(0,80)$

$\Longrightarrow$ x - intercept $= 30$
y - intercept $= 80$

Eq^n of line is

$$\frac{x}{30} + \frac{y}{80} = 1$$

$$\frac{80x + 30y}{2400} = 1$$

$$80x + 30y = 2400 \qquad \boxed{\text{i.e. } 8x + 3y = 240}$$

Equation of line	x-intercept	y-intercept
$3x+5y=90$	30	18
$5x-2y=200$	40	-100
$13x+18y=k$	$\frac{k}{13}$	$\frac{k}{18}$
$20x+13y=500$	25	$\frac{500}{13}$
$2x-11y=-53$	$-53/2$	$53/11$
$21x-y=200$	$200/21$	-200
$x-y=10$	10	-10
$2x+y=58$	29	58
$x=90$	90	No Y-intercept
$y=65$	No x intercept	65
$kx+my=j$	j/k	j/m
$2kx+3my=93$	$93/2k$	$93/3m = 31/m$
$x+2y=m$	m	$m/2$
$5x+3y=1500$	300	500
$x=90/7$	$90/7$	No Y-intercept

(73) Find Eq^n of line having x intercept as 3m & y intercept as 38.

$\Longrightarrow$

$$\frac{x}{3m} + \frac{y}{38} = 1$$

$$\frac{38x + 3my}{114m} = 1$$

$$38x + 3my = 114m$$

(74) Find Eq^n of the line having slope of $-8/11$ and x intercept as 12

$\Longrightarrow$ $8x + 11y = 96$

(75) Find slope of the line whose y intercept is 4 times of x intercept.

$\Longrightarrow$

x intercept $= a$

y intercept $= b = 4a$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{4x}{4a} + \frac{y}{4a} = 1$$

$$4x + y = 4a$$

Slope of the line $= -4/1 = -4$

(76) Find slope of the line whose x intercept is $(4/5)^{th}$ of y intercept.

$\Longrightarrow$ x intercept $= a = \frac{4}{5}b$

y intercept $= b$

Eq^n of the Line : $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{\frac{4b}{5}} + \frac{y}{b} = 1$$

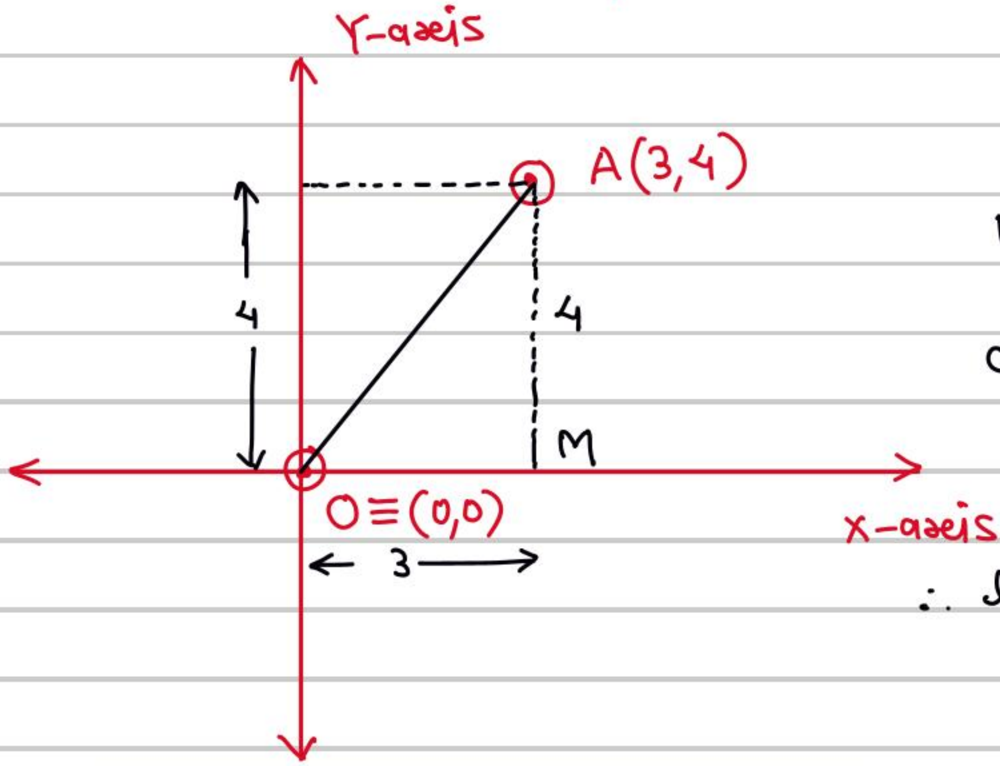
$$\frac{5x}{4b} + \frac{4y}{4b} = 1$$

$$\frac{5x + 4y}{4b} = 1$$

$$5x + 4y = 4b$$

slope of the line $= -5/4$

(77) If $A \equiv (3,4)$, $O \equiv (0,0)$ Find $l(OA) = ?$



ΔOAM is a Right angle triangle

$$OA^2 = OM^2 + AM^2$$

$$OA^2 = 3^2 + 4^2$$

$$\therefore l(OA) = \sqrt{3^2 + 4^2}$$

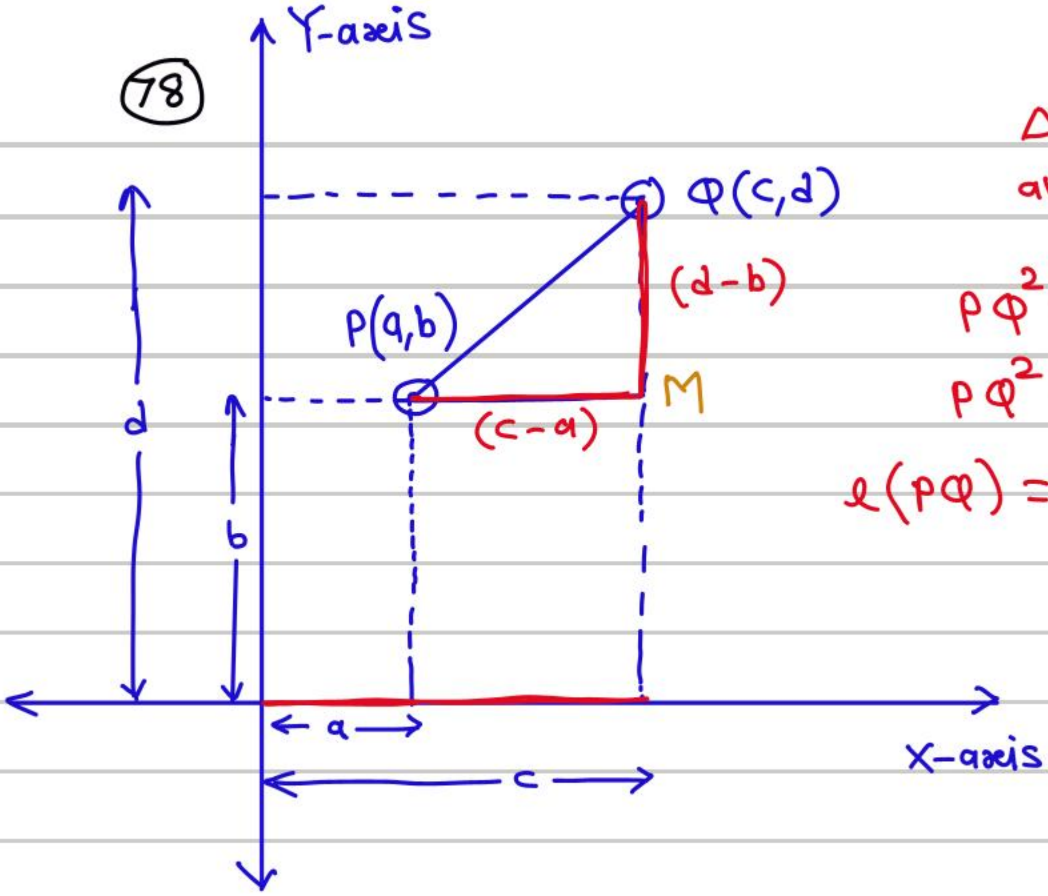
$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

If $O \equiv (0,0)$, $A \equiv (m,n)$
then $l(OA) = \sqrt{m^2 + n^2}$

78



ΔPQM is a Right angled triangle

$PQ^2 = PM^2 + QM^2$

$PQ^2 = (c-a)^2 + (d-b)^2$

$l(PQ) = \sqrt{(d-b)^2 + (c-a)^2}$

$P(a,b)$, $Q(c,d)$

then $l(PQ) = \sqrt{(d-b)^2 + (c-a)^2}$

$\therefore$ If $A \equiv (x_1, y_1)$ & $B \equiv (x_2, y_2)$ then

$$l(AB) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$$

If $A \equiv (x_1, y_1)$ & $B \equiv (0,0)$

then $l(AB) = \sqrt{x_1^2 + y_1^2}$

79 $A \equiv (30, 50)$, $B \equiv (80, -90)$ Find $l(AB)$

$\Longrightarrow$ $l(AB) = \sqrt{(-90-50)^2 + (80-30)^2}$

$= \sqrt{19600 + 2500} = 148.661$ units

(80) $P \equiv (m, n)$, $Q \equiv (i, j)$

then $l(PQ) = \sqrt{(j-n)^2 + (i-m)^2}$

$= \sqrt{(n-j)^2 + (m-i)^2}$

(81) If $A \equiv (1.50, 2.875)$, $B \equiv (33, 81.93)$

Find $l(AB)$

$\Longrightarrow$ $l(AB) = \sqrt{(81.93 - 2.875)^2 + (33 - 1.50)^2}$

$= 85.09961$ units

(82) If $A \equiv (0, 0)$, $B \equiv (-8.75, 33.8175)$

Find $l(AB)$

$\Longrightarrow$

$l(AB) = \sqrt{(33.8175 - 0)^2 + (-8.75 - 0)^2}$

$= 34.93112$ units

(83) points $(8, 13)$, $(16, 19)$, $(-2k, 48)$

are collinear. Find k.

$\Longrightarrow$ Points are said to be collinear if a straight line can pass through all of them

slope of a line passing through points $(8, 13)$, $(16, 19)$ = slope of the line passing through $(-2k, 48)$, $(16, 19)$

$$\frac{19 - 13}{16 - 8} = \frac{19 - 48}{16 - (-2k)}$$

$$\frac{6}{8} = \frac{-29}{16+2k}$$

$$6(16+2k) = -29 \times 8$$

$$96 + 12k = -232$$

$$12k = -328$$

$$k = -\frac{328}{12} = \left(-\frac{82}{3}\right)$$

$$k = -27.333333 = -27\frac{1}{3}$$

(84) points A, B, C are said to be collinear If

$$\left(\begin{array}{l}\text{slope of the line passing} \\ \text{through points A, B}\end{array}\right) = \left(\begin{array}{l}\text{slope of the line passing} \\ \text{through C \& A OR B}\end{array}\right)$$

(85) points $(16, -2k/5)$, $(8, 11)$, $(19, 85)$ are collinear. Find value of k.

$\Longrightarrow$

$$\frac{11 + \frac{2k}{5}}{8-16} = \frac{85-11}{19-8}$$

$$11 + \frac{2k}{5} = \frac{74}{11} \times -8$$

$$\boxed{k = -162.0454545}$$

(86) <u>Quadratic Equations</u>

The standard format of quadratic equation is:

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

and values of x which can satisfy the quadratic equation are known as '<u>roots</u>' of quadratic equation

$x^2 - 5x - 6 = 0$

In this quadratic eqn $a = 1,\ b = -5,\ c = -6$

If we put $x = 6$, $6^2 - 5(6) - 6 = 36 - 30 - 6 = 0$

If we put $x = -1$, $(-1)^2 - 5(-1) - 6 = 1 + 5 - 6 = 0$

$\therefore$ 6, −1 are roots of quadratic Equation $x^2 - 5x - 6 = 0$

(87)

Equation	No. of roots
Linear	1
quadratic	2
cubic	3

(88)

Quadratic Equation	a	b	c
$3x^2 + 5x - 8 = 0$	3	5	−8
$19x^2 - 55mx - 2k - 81 = 0$	19	$-55m$	$-2k - 81$ $= -(2k + 81)$
$15x^2 - 21x - 8px + 39x + 88k - 93 = 18x^2$ i.e. $-3x^2 + (-21 - 8p + 39)x + 88k - 93 = 0$	−3	$(-21 - 8p + 39)$ $= -(21 + 8p - 39)$	$88k - 93$
$10x^2 - 2p + 63 = 0$	10	0	$-2p + 63$
$55x^2 - kx^2 + 8px - 33mx + 18j = 63$ i.e. $(55 - k)x^2 + (8p - 33m)x + 18j - 63 = 0$	$55 - k$	$8p - 33m$	$18j - 63$
$17x^2 - 3x - 93 = 0$	17	−3	−93
$x^2 - 25 = 0$	1	0	−25
$x^2 = 58$	1	0	−58
$(p+q)x^2 - (p^2q^2x) - 33m = 80$	$(p+q)$	$-p^2q^2$	$-33m - 80$

(89) Find roots of Quadratic Equation

$$\boxed{x^2 - 13x + 36 = 0}$$

Formula Method

$x^2 - 13x + 36 = 0$

$a = 1,\ b = -13,\ c = 36$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{-13^2 - 4(1)(36)}}{2 \times 1}$$

$$x = \frac{13 \pm \sqrt{169 - 144}}{2}$$

$$x = \frac{13 \pm \sqrt{25}}{2} = \frac{13 \pm 5}{2}$$

$$x = \frac{13+5}{2} \text{ OR } x = \frac{13-5}{2}$$

$x = 9$ OR $x = 4$

$\therefore$ 4, 9 are roots of quadratic equation

Short-cut

$x^2 - 13x + 36 = 0$

First Find value of $ac = 36$ & $b = -13$

Find 2 numbers such that their sum is 'b' & their product is 'ac'

$x^2 - 9x - 4x + 36 = 0$

$x(x-9) - 4(x-9) = 0$

$(x-9)(x-4) = 0$

$x - 9 = 0$ OR $x - 4 = 0$

$x = 9$ OR $x = 4$

Super short cut

is applicable only when $a = 1$

↓

Find 2 numbers such that their sum is 'b' & product is 'c'

$(x-9)(x-4) = 0$

$x = 9$ OR $x = 4$

$\therefore$ 4, 9 are roots of quad. equation.

(90) Find roots of quadratic equation

$$5x^2 - 13x - 18 = 0$$

Formula

$$x = \frac{-(-13) \pm \sqrt{169 - 4(5)(-18)}}{2 \times 5}$$

$$x = \frac{13 \pm \sqrt{529}}{10} = \frac{13 \pm 23}{10}$$

$$x = \frac{13+23}{10} \text{ OR } x = \frac{13-23}{10}$$

$x = 3.60$ OR $x = -1.00$

short cut

$5x^2 - 13x - 18 = 0$

$5x^2 - 18x + 5x - 18 = 0$

$x(5x - 18) + 1(5x - 18) = 0$

$(5x - 18)(x + 1) = 0$

$\therefore x = 18/5$ OR $x = -1$

$\therefore$ Roots are : $18/5$ & -1

Factors are : $(5x-18), (x+1)$

(91) Find roots of Quadratic Equation

$$2x^2 + 21x + 9 = 0$$

$\Longrightarrow$

$$x = \left(\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$$

$$x = \frac{-21 \pm \sqrt{441-4(2)(9)}}{2 \times 2}$$

$$x = \frac{-21 \pm \sqrt{369}}{4}$$

$x = \dfrac{-21+\sqrt{369}}{4}$ OR $x = \dfrac{-21-\sqrt{369}}{4}$

$x = -0.447656$ OR $x = -10.0523$

(Roots are ir-rational Numbers)

(92) Find roots of : $x^2 - 11x - 102 = 0$

$\Longrightarrow$ $(x-17)(x+6) = 0$

$\therefore$ $x = 17$, $x = -6$

$\therefore$ Roots are : $17, -6$

(93) Find roots of $10x^2 - x - 24 = 0$

$\Longrightarrow$ $ac = -240$, $b = -1$

$10x^2 - x - 24 = 0$

$10x^2 - 16x + 15x - 24 = 0$

$2x(5x-8) + 3(5x-8) = 0$

$(5x-8)(2x+3) = 0$

$\therefore$ Roots are : $\frac{8}{5}$ & $-\frac{3}{2}$

(94) Find roots of quadratic Equation

$$80x^2 - 138x + 13 = 0$$

$\Longrightarrow$

$80x^2 - 138x + 13 = 0$

$80x^2 - 130x - 8x + 13 = 0$

$10x(8x - 13) - 1(8x - 13) = 0$

$(8x - 13)(10x - 1) = 0$

$x = 13/8$ OR $x = 1/10$

Roots are $\frac{13}{8}, \frac{1}{10}$

(i.e. 1.625, 0.10)

$x = \frac{-(-138) \pm \sqrt{19044 - 4(80)(13)}}{2 \times 80}$

$x = \frac{138 \pm \sqrt{14884}}{160} = \frac{138 \pm 122}{160}$

$x = \frac{138 + 122}{160}$ OR $x = \frac{138 - 122}{160}$

$x = \frac{260}{160}$ OR $x = \frac{16}{160}$

$x = \frac{13}{8}$ OR $x = \frac{1}{10}$

(95) Find Roots of quadratic Equation

$14x^2 + 29x - 15 = 0$.

Also Find sum of roots, product of roots.

$\Longrightarrow$

$14x^2 + 29x - 15 = 0$

$14x^2 + 35x - 6x - 15 = 0$

$7x(2x + 5) - 3(2x + 5) = 0$

$(2x + 5)(7x - 3) = 0$

$\therefore$ $x = -5/2$ OR $x = 3/7$

Roots are $-\frac{5}{2}$ & $\frac{3}{7}$

Sum of roots

$= 1^{st}$ root $+ 2^{nd}$ root

$= \frac{-5}{2} + \frac{3}{7}$

$= \frac{-35 + 6}{14} = \frac{-29}{14}$

product of roots

$= 1^{st}$ root $\times 2^{nd}$ root

$= -\frac{5}{2} \times \frac{3}{7} = -\frac{15}{14}$

For a quadratic Equation

sum of roots $= -b/a$

product of roots $= c/a$

(96) Find roots of quadratic equation

$10x^2 - 59x - 6 = 0$.

Also find sum of roots, product of roots.

$\Longrightarrow$

$$10x^2 - 59x - 6 = 0$$
$$10x^2 - 60x + x - 6 = 0$$
$$10x(x-6) + 1(x-6) = 0$$
$$(x-6)(10x+1) = 0$$
$$\therefore x = 6 \text{ OR } x = -1/10$$

Roots are : 6 & $-1/10$

sum of roots

$= 1^{st}$ root $+ 2^{nd}$ root

$= 6 + \left(-\frac{1}{10}\right)$

$= \frac{59}{10} = (59/10) = 5.90$

product of roots

$= 1^{st}$ root $\times 2^{nd}$ root

$= 6 \times -1/10 = -6/10$

$= -3/5$

(97)

Quadratic Equation	sum of roots	product of roots
$ax^2 + bx + c = 0$	$-b/a$	c/a
$8x^2 - 15x - 33 = 0$	$15/8$	$-33/8$
$2x^2 - px + mq + 93 = 0$	$p/2$	$(mq+93)/2$
$x^2 - 40 = 0$	0	$-40/1 = -40$
$px^2 + qx + r = 0$	$-q/p$	r/p
$(3k+3)x^2 - (2p-q)x + 8j + 63 = 0$	$\frac{(2p-q)}{(3k+3)}$	$\left(\frac{8j+63}{3k+3}\right)$

98 In a quadratic eqn $ax^2 + bx + c = 0$

① sum of roots = 1st root + 2nd root

$$= \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) + \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \left(\frac{-b+\cancel{\sqrt{b^2-4ac}}-b-\cancel{\sqrt{b^2-4ac}}}{2a}\right)$$

$$= \left(\frac{-2b}{2a}\right) = -b/a$$

② product of roots = 1st root × 2nd root

$$= \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) \times \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \left[\frac{\left(-b+\sqrt{b^2-4ac}\right) \times \left(-b-\sqrt{b^2-4ac}\right)}{4 \times a \times a}\right]$$

$$= \left[\frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{4 \times a \times a}\right] = \left[\frac{b^2-(b^2-4ac)}{4a \times a}\right]$$

$$= \left[\frac{\cancel{b^2}-\cancel{b^2}+4ac}{4a \times a}\right] = \frac{\cancel{4a}c}{\cancel{4a} \times a}$$

$$= c/a$$

(99) If α, β are roots of quadratic Eqn

$5x^2 - 3x - 8 = 0$

Find the values of $(\alpha+\beta)$, $\alpha\beta$, $(\alpha+\beta)^2$, $(\alpha^2+\beta^2)$

$\Longrightarrow$ ① $\alpha + \beta$ = sum of roots = $3/5$

② $\alpha\beta$ = product of roots = $-8/5$

③ $(\alpha+\beta)^2 = (3/5)^2 = 9/25$

④ $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$= \frac{9}{25} - 2\left(-\frac{8}{5}\right) = \frac{9}{25} + \frac{16}{5}$$

$$= \frac{9}{25} + \frac{80}{25} = \left(89/25\right)$$

$$\boxed{\begin{aligned} \alpha^2+\beta^2 &= (\alpha+\beta)^2 - 2\alpha\beta \\ \alpha^3+\beta^3 &= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) \\ (\alpha-\beta)^2 &= (\alpha+\beta)^2 - 4\alpha\beta \end{aligned}}$$

(100) If p, q are roots of quadratic equation

$x^2 - 3x + 20 = 0$. Find values of

i) $p+q = 3$

ii) $pq = 20$

iii) $(p-q)^2 = (p+q)^2 - 4pq = 9 - 4(20) = -71$

iv) $(p^2+q^2) = (p+q)^2 - 2pq = 9 - 2(20) = -31$

v) $p^3+q^3 = (p+q)^3 - 3pq(p+q) = 3^3 - 3(20)(3) = 27 - 180$
$= -153$

vi) $p^2q + q^2p = pq(p+q) = 20 \times 3 = 60$

(101) If α, β are roots of Quadratic Equation $3x^2 - 5x + 2 = 0$. Find values of

i) $\alpha + \beta = 5/3$

ii) $\alpha\beta = 2/3$

iii) $(\alpha+\beta)^3 = (5/3)^3 = 125/27$

iv) $(\alpha^2+\beta^2) = (\alpha+\beta)^2 - 2\alpha\beta = \frac{25}{9} - 2\left(\frac{2}{3}\right) = \frac{25}{9} - \frac{12}{9} = \frac{13}{9}$

v) $(\alpha^3+\beta^3) = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \left(\frac{125}{27} - 3 \times \frac{2}{3} \times \frac{5}{3}\right)$

$= \left(\frac{125}{27} - \frac{30}{9}\right) = \left(\frac{125}{27} - \frac{90}{27}\right) = 35/27$

vi) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = \frac{25}{9} - 4\left(\frac{2}{3}\right) = \frac{25}{9} - \frac{8}{3}$

$= \frac{25}{9} - \frac{24}{9} = \frac{1}{9}$

vii) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta)$

$= \frac{2}{3} \times \frac{5}{3} = (10/9)$

viii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{13/9}{2/3} = \frac{13/9}{6/9} = 13/6$

ix) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \left(\frac{35/27}{2/3}\right) = \left[\frac{35}{\cancel{27}\,9} \times \frac{\cancel{3}}{2}\right] = (35/18)$

(102) If α, β are roots of Quadratic Equation $x^2 + 5x + 13 = 0$. Find values of

(a) $\alpha + \beta = -5$

(b) $\alpha\beta = 13$

(c) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 25 - 2(13) = -1$

(d) $(\alpha+\beta)^2 = (-5)^2 = 25$

(e) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = 25 - 4(13) = -27$

(f) $\alpha^2\beta^2 = (\alpha\beta)^2 = 13^2 = 169$

(g) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta) = 13 \times -5 = -65$

(h) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \left(\dfrac{\alpha^2+\beta^2}{\alpha\beta}\right) = \dfrac{-1}{13}$

(i) $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} = \left(\dfrac{\alpha^3+\beta^3}{\alpha\beta}\right) = \left(\dfrac{70}{13}\right)$

(j) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -125 - 3(13)(-5)$

$= -125 + 195$

$= 70$

(103) The standard format of Quadratic Equation is,

$ax^2 + bx + c = 0$ where $a \neq 0$

$ax^2 - (-b)x + c = 0$

dividing by 'a' on both sides

$\dfrac{ax^2}{a} - \left(\dfrac{-b}{a}\right)x + \left(\dfrac{c}{a}\right) = 0/a$

$x^2 - \left(-\dfrac{b}{a}\right)x + \left(\dfrac{c}{a}\right) = 0$

$\boxed{x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0}$

(104)

Find Roots of quadratic Eqn $10x^2+11x+1=0$

$\Rightarrow$ $10x^2+10x+x+1=0$

$10x(x+1)+1(x+1)=0$

$(x+1)(10x+1)=0$

$\therefore$ $x=-1$ OR $x=-1/10$

Roots are : $-1, -1/10$

Find the quadratic equation whose roots are $-1, -1/10$

$\Rightarrow$ $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$x^2-\left(-1+\frac{-1}{10}\right)x+\left(-1\times-\frac{1}{10}\right)=0$

$x^2-\left(\frac{-11}{10}\right)x+\left(\frac{1}{10}\right)=0$

$10x^2+11x+1=0$

(105) Find roots of quadratic Eqn $16x^2+36x-10=0$

$\Rightarrow$ $16x^2+36x-10=0$

$8x^2+18x-5=0$

$8x^2+20x-2x-5=0$

$4x(2x+5)-1(2x+5)=0$

$(2x+5)(4x-1)=0$

$x=-5/2$, $x=1/4$

Roots are : $-5/2$ & $1/4$

Find quadratic Eqn whose roots are $-5/2$ & $1/4$

$\Rightarrow$ $x^2-\left(\text{sum of roots}\right)x+\left(\text{product of roots}\right)=0$

$x^2-\left(\frac{-5}{2}+\frac{1}{4}\right)x+\left(\frac{-5}{2}\times\frac{1}{4}\right)=0$

$x^2-\left(\frac{-18}{8}\right)x-\frac{5}{8}=0$

$8x^2+18x-5=0$

$16x^2+36x-10=0$

(106) Find roots of quadratic Equation $6x^2+19x-7=0$

$\Rightarrow$ $6x^2+19x-7=0$

$6x^2+21x-2x-7=0$

$3x(2x+7)-1(2x+7)=0$

$(2x+7)(3x-1)=0$

Roots are : $-7/2$ & $1/3$

Find quadratic equation whose roots are $-7/2$ & $1/3$

$\Rightarrow$ $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$x^2-\left(\frac{1}{3}-\frac{7}{2}\right)x+\left(\frac{1}{3}\times-\frac{7}{2}\right)=0$

$x^2-\frac{-19}{6}x-\frac{7}{6}=0$

$6x^2+19x-7=0$

107 Find roots of quadratic Eqn $x^2-10x+23=0$

$\Rightarrow$ $x = \left(\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$

$x = \dfrac{10 \pm \sqrt{100-4(1)(23)}}{2\times 1}$

$x = \dfrac{10\pm\sqrt{8}}{2} = \dfrac{10\pm\sqrt{4\times 2}}{2}$

$x = \dfrac{10 \pm 2\sqrt{2}}{2} = \dfrac{2(5\pm\sqrt{2})}{2} = 5\pm\sqrt{2}$

Roots are : $(5+\sqrt{2})$ & $(5-\sqrt{2})$

Find quadratic Eqn whose roots are $(5+\sqrt{2})$ & $(5-\sqrt{2})$

$\Rightarrow$

$x^2 - \left(\text{sum of roots}\right)x + \left(\text{product of roots}\right) = 0$

$x^2 - (5+\cancel{\sqrt{2}}+5-\cancel{\sqrt{2}})x + \left[(5+\sqrt{2})\times(5-\sqrt{2})\right] = 0$

$x^2 - 10x + (5^2-2) = 0$

$x^2 - 10x + 23 = 0$

108

Roots of quadratic Eqn	Quadratic Equation
5, 10	$x^2-15x+50=0$
−18, 20	$x^2-2x-360=0$
1, −1	$x^2-0x-1=0$ i.e. $x^2-1=0$
15, 18	$x^2-33x+270=0$
−16, −20	$x^2+36x+320=0$
$-5/2$, $9/2$	$x^2-\frac{4}{2}x+\frac{-45}{4}=0$ $4x^2-8x-45=0$
$9/7$, $8/13$	$x^2-\frac{173}{91}x+\frac{72}{91}=0$ $91x^2-173x+72=0$
16, 0	$x^2-16x+0=0$ i.e. $x^2-16x=0$
$(8+\sqrt{3})$, $(8-\sqrt{3})$	$x^2-16x+(64-3)=0$ i.e. $x^2-16x+61=0$
$(1+\sqrt{30})$, $(1-\sqrt{30})$	$x^2-2x+(1-30)=0$ i.e. $x^2-2x-29=0$

(109) Find roots of quadratic Equation

$4x^2+12x+9=0$

$\Longrightarrow$

$4x^2+12x+9=0$

$4x^2+6x+6x+9=0$

$2x(2x+3)+3(2x+3)=0$

$(2x+3)(2x+3)=0$

$2x+3=0$ OR $2x+3=0$

$x=-3/2$ OR $x=-3/2$

$\therefore$ Roots are : $-3/2$ & $-3/2$

b^2-4ac

$=12^2-4(4)(9)$

$=144-144$

$=0$

Roots of this quadratic Equation are Equal.

When $b^2-4ac=0$ then Roots of quadratic Equation are Equal

(110) Find roots of quadratic eqn

$5x^2+15x+91=0$

$\Longrightarrow$

$$x=\left(\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right)$$

$$x=\frac{-15\pm\sqrt{225-4(5)(91)}}{2\times5}=\frac{-15\pm\sqrt{-1595}}{10}$$

Roots of quadratic equation are complex/Imaginary/unreal

When $b^2-4ac<0$ then roots of quadratic Equation are complex/unreal/Imaginary

(111) Find roots of Quadratic Equation

$$x^2 - 14x + 46 = 0$$

$\Longrightarrow$

$$x = \left[\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(46)}}{2 \times 1}$$

$$x = \frac{14 \pm \sqrt{12}}{2} = \frac{14 \pm 2\sqrt{3}}{2}$$

$$x = \frac{2(7 \pm \sqrt{3})}{2} = 7 \pm \sqrt{3}$$

Roots are $(7+\sqrt{3})$, $(7-\sqrt{3})$

Roots are irrational Numbers

When $b^2 - 4ac > 0$ but not a perfect square then Roots of Quadratic Equation are irrational

112

When	Nature of roots
$b^2 - 4ac = 0$	Real, Rational, Equal
$b^2 - 4ac < 0$	Unreal, complex, imaginary
$b^2 - 4ac > 0$ and Not a perfect square	Real, irrational, unequal
$b^2 - 4ac > 0$ and a perfect square	Real, Rational, unequal

(113) Find the quadratic Eq^n whose roots are $3/2$, $-8/11$

$\Rightarrow$ $x^2 - (\text{Sum of roots})x + \begin{pmatrix}\text{Product}\\ \text{of roots}\end{pmatrix} = 0$

$x^2 - \left(\frac{3}{2} - \frac{8}{11}\right)x + \left(\frac{3}{2} \times \frac{-8}{11}\right) = 0$

$x^2 - \frac{17}{22}x - \frac{24}{22} = 0$

$22x^2 - 17x - 24 = 0$

(114) Find the quadratic eq^n whose roots are $(2 + \sqrt{23})$ & $(2 - \sqrt{23})$

$\Rightarrow$ $x^2 - (2 + \cancel{\sqrt{23}} + 2 - \cancel{\sqrt{23}})x + [(2 + \sqrt{23})(2 - \sqrt{23})] = 0$

$x^2 - 4x - 19 = 0$

(115) Find the quadratic whose one root is $(15+\sqrt{41})$

$\Rightarrow$ If one root is $(15+\sqrt{41})$ then other root must be $(15-\sqrt{41})$

Roots are: $(15+\sqrt{41}), (15-\sqrt{41})$

Quadratic eqn is

$$x^2 - 30x + (225-41) = 0$$

$$x^2 - 30x + 184 = 0$$

(116)

Quadratic Eqn	b^2-4ac	Nature of roots
$3x^2-5x-8=0$	$-5^2-4(3)(-8)$ $= 25+96 = 121$	Real, rational, unequal
$8x^2-13x+200=0$	$-13^2-4(8)(200)$ $= 169-6400$ $= -6231$	complex/imaginary OR unreal
$5x^2+11x-3=0$	$11^2-4(5)(-3)$ $= 121+60 = 181$	Real, irrational, unequal
$4x^2+12x+9=0$	$12^2-4(4)(9)$ $= 144-144 = 0$	Real, Rational, Equal
$x^2-13x+36=0$	$-13^2-4(1)(36)$ $= 169-144 = 25$	Real, Rational, Distinct
$5x^2+12x+7=0$	$12^2-4(5)(7)$ $= 4$	Real, Rational, unequal
$4x^2-1=0$	$0^2-4(4)(-1)$ $= 0+16 = 16$	Real, Rational, unequal
$3x^2+22x=0$	$22^2-4(3)(0)$ $= 484$	Real, Rational, unequal
$8x^2-2x+33=0$	$(-2)^2-4(8)(33) = -1052$	complex/imaginary

(117)

Value of b^2-4ac	Nature of roots
38	Real, irrational, unequal
41	Real, irrational, unequal
49	Real, Rational, unequal
−60	complex / imaginary / unreal
0	Real, Rational, Equal
88	Real, irrational, unequal
14641	Real, Rational, unequal
19288	Real, irrational, unequal
3364	Real, Rational, unequal
−0	Real, Rational, Equal
380	Real, irrational, unequal
−100	complex / Imaginary / unreal

(118) Roots of quadratic Eq^n $5x^2-33x+8k+5=0$ are Equal. Find k.

$\Longrightarrow$ $a=5,\ b=-33,\ c=8k+5$

As roots are equal, $b^2-4ac=0$

$-33^2-4(5)(8k+5)=0$

$1089-160k-100=0$

$989=160k$

$\therefore\ k=6.18125$

(119) Roots of quadratic Eq^n $5kx^2-3x^2+18x-21=0$ are equal. Find k.

$\Longrightarrow$ $(5k-3)x^2+18x-21=0$

$b^2-4ac=0$

$18^2-4(5k-3)(-21)=0$

$324+84(5k-3)=0$

$\therefore\ k=-0.17142857$

(120) Roots of quadratic Equation

$5mx^2 + 33x - 28 = 0$ are equal. Find m.

$\Rightarrow$ $b^2 - 4ac = 0$

$33^2 - 4(5m)(-28) = 0$

$1089 + 560m = 0$

$560m = -1089$

$\boxed{m = -1.9446}$

(121) Roots of quadratic Equation

$5kx^2 - 33x + 8k - 19 = 0$ are reciprocals of each other. Find value of k.

$\Rightarrow$ As Roots are Reciprocals of each other

1st root × 2nd root = 1

product of roots = 1

$c/a = 1$

$c = a$

$a = c$

$\therefore\ 5k = 8k - 19$

$19 = 3k$

$\therefore\ k = 19/3 = 6.33333 = 6\frac{1}{3}$

(122) Roots of quadratic Eq^n

$5x^2 - 8kx + 33x - 8p - 19 = 0$

are Equal but opposite in sign. Find value of k.

$\Rightarrow$ As Roots are Equal but opposite in sign,

1st root + 2nd root = 0

$\therefore$ Sum of roots $= 0$

$$\frac{-b}{a} = 0$$

$$-b = 0$$

$$b = 0$$

Here $(-8k + 33) = 0$

$$33 = 8k$$

$$\therefore\ k = 33/8 = 4.1250$$

(123)

If Roots of quadratic Equation are	then
Equal	$b^2 - 4ac = 0$
Reciprocals of each other	$a = c$
Equal but opposite in sign	$b = 0$ $= \text{ZERO}$

(124) Roots of quadratic eqn $5x^2 + kx^2 - 19x - 33k - 93 = 0$ are reciprocals of each other. Find k.

$\Longrightarrow$ $(5+k)x^2 - 19x - 33k - 93 = 0$

As Roots are reciprocals of each other

$$a = c$$

$$5+k = -33k - 93$$

$$98 = -34k$$

$$k = -98/34 = -49/17$$

$$k = -2.8824$$

(125) Roots of quadratic eqn $5x^2 - 8px + 81x = 93x - 63k + 88$ are equal but opposite in sign. Find p.

$$\Rightarrow 5x^2 - 8px + 81x - 93x + 63k - 88 = 0$$

$$5x^2 + (-8p + 81 - 93)x + 63k - 88 = 0$$

$$5x^2 + (-8p - 12)x + 63k - 88 = 0$$

$\therefore \quad b = 0$ ------ As Roots are equal but opposite in sign.

$$-8p - 12 = 0$$

$$-12 = 8p$$

$$\therefore p = -12/8 = -3/2 = -1.50$$

(126) If p, q are roots of quadratic equation $x^2 - 11x - 28 = 0$ Find values

1. $p + q$ = sum of roots = 11
2. pq = product of roots = -28
3. $p^3 + q^3 = (p+q)^3 - 3pq(p+q) = 1331 - 3(-28)(11) = 2{,}255$
4. $p^2 + q^2 = (p+q)^2 - 2pq = 121 - 2(-28) = 177$
5. $(p-q)^2 = (p+q)^2 - 4pq = 121 - 4(-28) = 233$
6. $\dfrac{p}{q} + \dfrac{q}{p} = \dfrac{p^2 + q^2}{pq} = \dfrac{177}{-28} = -\left(177/28\right)$
7. $\dfrac{p^2}{q} + \dfrac{q^2}{p} = \dfrac{p^3 + q^3}{pq} = \dfrac{2255}{-28} = -\left(2255/28\right) = -80.5357$
8. $p^2q + q^2p = pq(p+q) = -28 \times 11 = -308$
9. $(p-q) = \sqrt{(p-q)^2} = \sqrt{233} = \pm 15.2643375$
10. $p^2q^2 = (pq)^2 = (-28)^2 = 784$

(127) If α, β are roots of quadratic equation $5x^2 - 2x + 3 = 0$. Find Quadratic equation whose roots are $(\alpha^2+\beta^2)$, $(\alpha-\beta)^2$

$\Longrightarrow$ $\alpha+\beta = 2/5$, $\alpha\beta = 3/5$

$$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{4}{25} - \frac{30}{25} = -26/25$$

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = \frac{4}{25} - \frac{60}{25} = -56/25$$

$\therefore$ question is : Find quad. eq^n whose roots are $-26/25$ & $-56/25$

$$x^2 - \left(\frac{-56}{25} + \frac{-26}{25}\right)x + \left(\frac{-26}{25} \times \frac{-56}{25}\right) = 0$$

$$x^2 - \frac{-82}{25}x + \frac{1456}{625} = 0$$

$$625x^2 + 2050x + 1456 = 0$$

(128) If α, β are roots of quad. eq^n $x^2 - 12x + 17 = 0$. Find quad. eq^n whose roots are $(\alpha^3+\beta^3)$, $(\alpha^2+\beta^2)$

$\Longrightarrow$ $\alpha+\beta = 12$

$\alpha\beta = 17$

$$\therefore (\alpha^3+\beta^3) = 12^3 - (3\times17\times12) = 1728 - 612 = 1116$$

$$(\alpha^2+\beta^2) = 12^2 - 2(17) = 110$$

$\therefore$ quad. eq^n whose roots are 1116 & 110 is

$$x^2 - (1116+110)x + (1116\times110) = 0$$

$$x^2 - (1226)x + (1,22,760) = 0$$

(129) If α, β are roots of quad. eqn

$5x^2 - 2x - 11 = 0$

Find quad. eqn whose roots are $(\alpha+\beta)$, $(\alpha\beta)$

$\Longrightarrow$ $\alpha + \beta = 2/5$ $\quad \alpha\beta = -11/5$

$\therefore$ The question is: Find the quad. eqn whose roots $2/5$ & $-11/5$

$$x^2 - \left(-\frac{9}{5}\right)x + \left(\frac{-22}{25}\right) = 0$$

$\therefore$ $25x^2 + 45x - 22 = 0$

(130) If $a+b = 12$, $ab = 60$ Find $\left(1/a + 1/b\right) = ?$

$$\Longrightarrow \frac{1}{a} + \frac{1}{b} = \left(\frac{a+b}{ab}\right) = \left(\frac{12}{60}\right) = 1/5 = 0.20$$

(131) Find quadratic eqn whose one root is $(11 + \sqrt{13})$

$\Longrightarrow$ If one root is $(11+\sqrt{13})$ then other must be $(11-\sqrt{13})$

Roots are: $(11+\sqrt{13})$, $(11-\sqrt{13})$

Quadratic Eqn is : $x^2 - 22x + 108 = 0$

(132) Find the quadratic eqn whose one root is $(7 + \sqrt{230})$

$\Longrightarrow$ Roots are: $(7+\sqrt{230})$ & $(7-\sqrt{230})$

$$x^2 - 14x + (-181) = 0$$

$$x^2 - 14x - 181 = 0$$

(133) Standard format of a quadratic Eq^n is :

$$ax^2 + bx + c = 0 \qquad x^2 - \begin{pmatrix}\text{sum of}\\ \text{roots}\end{pmatrix}x + \begin{pmatrix}\text{product}\\ \text{of roots}\end{pmatrix} = 0$$

where $a \neq 0$

(134)

Roots of quadratic eq^n	Factors of quadratic eq^n	quadratic Eq^n
$3, -13$	$(x-3)(x+13)$	$x^2+10x-39=0$
$-3/2, 1/8$	$(2x+3)(8x-1)$	$16x^2+22x-3=0$
$1/2, 1/2$	$(2x-1)(2x-1)$	$4x^2-4x+1=0$
$2/5, 9/8$	$(5x-2), (8x-9)$	$40x^2-61x+18=0$
$-3/5, 7/11$	$(5x+3), (11x-7)$	$55x^2-2x-21=0$
$-4/3, -1/2$	$(3x+4)(2x+1)$	$6x^2+11x+4=0$
$7/5, -11/8$	$(5x-7), (8x+11)$	$40x^2-x-77=0$
$0, 8$	$x, (x-8)$	$x^2-8x=0$
$1, -1$	$(x-1), (x+1)$	$x^2-1=0$
$5/3, 3/5$	$(3x-5), (5x-3)$	$15x^2-34x+15=0$

(135) Standard format of a cubic eqn

$$ax^3 + bx^2 + cx + d = 0$$

where $a \neq 0$

3 values of x can satisfy cubic Equation

$\therefore$ cubic equation has 3 roots.

(136) Find cubic eqn whose roots are $8, 3, -2$

$\Longrightarrow$ As roots are $8, 3, -2$

Factors must be $(x-8), (x-3), (x+2)$

$\therefore$ cubic eqn is:

$$(x-8)(x-3)(x+2) = 0$$

$$(x^2 - 11x + 24)(x+2) = 0$$

$$x^3 + 2x^2 - 11x^2 - 22x + 24x + 48 = 0$$

$$x^3 - 9x^2 + 2x + 48 = 0$$

(137) Find the cubic equation whose roots are p, q, r

$\Longrightarrow$ Factors of cubic eqn are: $(x-p)(x-q)(x-r)$

$\therefore$ cubic eqn is:

$$(x-p)(x-q)(x-r) = 0$$

$$(x^2 - qx - px + pq)(x-r) = 0$$

$$x^3 - x^2 r - x^2 q + qrx - x^2 p + prx + pqx - pqr = 0$$

$$x^3 - x^2 r - x^2 q - x^2 p + qrx + prx + pqx - pqr = 0$$

$$x^3 - (p+q+r)x^2 + (pq + qr + pr)x - (pqr) = 0$$

$$x^3 - (\text{sum of roots})x^2 + \left[(1^{st} \times 2^{nd}) + (2^{nd} \times 3^{rd}) + (1^{st} \times 3^{rd})\right]x - \left(\text{product of roots}\right) = 0$$

Let's compare this with

$$ax^3 + bx^2 + cx + d = 0$$

$\therefore$ sum of roots $= -b/a$, product of roots $= -d/a$

(138) ① Find cubic eqn whose roots are

3, −11, −15

$\Longrightarrow$

$x^3 - (3-11-15)x^2 + [(3\times-11) + (-11\times-15) + (3\times-15)]x - (3\times-11\times-15) = 0$

$x^3 - (-23)x^2 + (-33+165-45)x - (495) = 0$

$x^3 + 23x^2 + 87x - 495 = 0$

OR

$(x-3)(x+11)(x+15) = 0$

$(x^2+8x-33)(x+15) = 0$

$x^3 + 15x^2 + 8x^2 + 120x - 33x - 495 = 0$

$x^3 + 23x^2 + 87x - 495 = 0$

② Find cubic eqn whose roots are m, n, v

$x^3 - (m+n+v)x^2 + (mn+nv+mv)x - mnv = 0$

(139)

	cubic Equation	quadratic Equation
standard format	$ax^3+bx^2+cx+d=0$ where $a\neq0$	$ax^2+bx+c=0$ where $a\neq0$
sum of roots	$-b/a$	$-b/a$
product of roots	$-d/a$	c/a

(140) Find cubic eqn whose are

$\left(5/2, 9/2, -11/2\right)$

$$\Rightarrow x^3 - \left(\frac{5}{2} + \frac{9}{2} + \frac{-11}{2}\right)x^2 + \left(\frac{45}{4} + \frac{-99}{4} + \frac{-55}{4}\right)x - \left(\frac{5}{2} \times \frac{9}{2} \times \frac{-11}{2}\right) = 0$$

$$x^3 - \frac{3}{2}x^2 + \frac{-109}{4}x + \frac{495}{8} = 0$$

$$8x^3 - 12x^2 - 218x + 495 = 0$$

OR

$$(2x-5)(2x-9)(2x+11) = 0$$

(141) Find quadratic eqn whose roots are

$5/2, 0$

$\Rightarrow$ Factors : $(2x-5), x$

quadratic eqn : $2x^2 - 5x = 0$

(142) Find quadratic eqn whose roots are

$10, -10$

$$\Rightarrow x^2 - (10 + -10)x + (10 \times -10) = 0$$

$$x^2 - 0x + (-100) = 0$$

$$x^2 - 100 = 0$$

(143) Find quadratic eqn whose roots are

$8/9, 9/8$

$$\Rightarrow x^2 - \left(\frac{8}{9} + \frac{9}{8}\right)x + \left(\frac{8}{9} \times \frac{9}{8}\right) = 0$$

$$x^2 - \frac{145}{72}x + 1 = 0$$

$$\boxed{\therefore 72x^2 - 145x + 72 = 0}$$

(144) The point $(-3p, 28)$ lie on the line $7x + 12y = 820$. Find p.

$\Longrightarrow$

$$7x + 12y = 820$$
$$7(-3p) + 12(28) = 820$$
$$-21p + 336 = 820$$
$$\boxed{p = -23.047619}$$

(145) The point of intersection of lines $7x + 3y = 90$ and $8x + 7y = 210$ lie in ______ quadrant.

(a) 1st (b) 2nd (c) 4th ~~(d)~~ None of these

$\Longrightarrow$

$$49x + \cancel{21y} = \cancel{630}$$
$$24x + \cancel{21y} = \cancel{630}$$
$$- \quad - \quad -$$
$$25x = 0$$
$$x = 0$$

$\therefore 7x + 3y = 90$

$7(0) + 3y = 90$

$y = 30$

$\therefore$ point of intersection $\equiv (0, 30)$

(146) The lines $2x + 3y = 90$ & $4x + 6y = 180$ have ______

(a) No solution

(b) unique solution

~~(c)~~ infinite no. of solutions

(d) None of these

$2x + 3y = 90$

$4x + 6y = 180$ ----- (1)

$4x + 6y = 180$ ----- (2)

As these 2 lines coincide

$\therefore$ They intersect in each point of line

(147) slope of the line $8x = 81/11$ is

(a) zero (b) $81/88$ (c) $81/-88$ ~~(d)~~ Not defined

$$8x = \frac{81}{11}$$
$$x = \frac{81}{88}$$

$\therefore x + 0y = \frac{81}{88}$

Slope $= -1/0$ = Not defined

(148) Find Eq^n of line having slope of $8/11$ passing through $(3/5, 8/5)$

$\Longrightarrow$ slope $= 8/11$

Eq^n of the line

$$8x - 11y = 8\left(\frac{3}{5}\right) - 11\left(\frac{8}{5}\right)$$

$$8x - 11y = -64/5$$

$$40x - 55y = -64$$

(149) Find Eq^n of the line having x, y intercept as $8/3$, $11/9$ resp.

$$\Longrightarrow \quad \frac{x}{8/3} + \frac{y}{11/9} = 1$$

$$\frac{3x}{8} + \frac{9y}{11} = 1$$

$$\frac{33x + 72y}{88} = 1$$

$$33x + 72y = 88$$

(150) Find Nature of roots of $3x^2 - 14x - 31 = 0$

$$\Longrightarrow \quad b^2 - 4ac = (-14)^2 - 4(3)(-31)$$

$$= 196 + 372$$

$$= 568$$

$\therefore$ Roots are Real, Irrational, unequal

(151) If α, β are roots of quadratic Eq^n $x^2 - 5x + 9 = 0$ then Find the quadratic Eq^n whose roots are $(2\alpha + 3\beta)$ & $(3\alpha + 2\beta)$

$\Longrightarrow$ $\alpha + \beta = 5$, $\alpha\beta = 9$

quadratic Eq^n whose roots are $(2\alpha + 3\beta)$ & $(3\alpha + 2\beta)$ is

$$x^2 - (2\alpha + 3\beta + 3\alpha + 2\beta)x + [(2\alpha + 3\beta)(3\alpha + 2\beta)] = 0$$

$$x^2 - (5\alpha + 5\beta)x + [6\alpha^2 + 13\alpha\beta + 6\beta^2] = 0$$

$$x^2 - 5(\alpha + \beta)x + [6(\alpha^2 + \beta^2) + 13\alpha\beta] = 0$$

$$x^2 - (5 \times 5)x + [6(25 - 18) + 13 \times 9] = 0$$

$$x^2 - 25x + (42 + 117) = 0$$

$$x^2 - 25x + 159 = 0$$

(152) one root of quadratic equation $3kx^2 + 18px - 19p + 21 = 0$ is 'zero'. Find value of 'p'.

$\Longrightarrow$ $3kx^2 + 18px - 19p + 21 = 0$

As '0' is one of the root, If we put $x = 0$ then quadratic eq^n must be satisfied

$\therefore$ Let's put $x = 0$

$$3k(0)^2 + 18p(0) - 19p + 21 = 0$$

$$(3k \times 0) + 0 - 19p + 21 = 0$$

$$0 + 0 - 19p = -21$$

$$-19p = -21$$

$$p = \frac{-21}{-19} = \left(\frac{21}{19}\right)$$

(153) If α, β are roots of $5x^2 - 11x + 29 = 0$, Find quadratic equation whose roots are $(\alpha+1)$ & $(\beta+1)$

$\Longrightarrow \quad \alpha + \beta = 11/5, \; \alpha\beta = 29/5$

quad. eqn whose roots are $(\alpha+1)$ & $(\beta+1)$ is,

$$x^2 - (\alpha+1+\beta+1)x + \left[(\alpha+1)(\beta+1)\right] = 0$$

$$x^2 - (\alpha+\beta+2)x + \left[\alpha\beta+\alpha+\beta+1\right] = 0$$

$$x^2 - \left(\frac{11}{5}+\frac{10}{5}\right)x + \left[\frac{29}{5}+\frac{11}{5}+\frac{5}{5}\right] = 0$$

$$x^2 - \frac{21}{5}x + \frac{45}{5} = 0 \qquad \therefore 5x^2 - 21x + 45 = 0$$

(154) The points $(16, -2k/9)$, $(18, 0)$, $(19, -23)$ are collinear. Find 'k'.

$$\Longrightarrow \quad \frac{0 + \frac{2k}{9}}{18-16} = \frac{-23-0}{19-18}$$

$$\frac{2k/9}{2} = -23$$

$$2k/9 = -46$$

$$2k = -414$$

$$k = -\frac{414}{2}$$

$$k = -207$$

(155) $\frac{x+24}{5} = 4 + \frac{x}{4}$. Find x.

$\Longrightarrow$ $\frac{x+24}{5} = 4 + \frac{x}{4}$

$\frac{x+24}{5} = \frac{16+x}{4}$

$4x + 96 = 80 + 5x$

$16 = x$

$\therefore x = 16$

(156) $x + 5y = 36$, $\frac{x+y}{x-y} = \frac{5}{3}$ then $(x, y) = ?$

$\Longrightarrow$

$x + 5y = 36$

$x = 36 - 5y$

$\frac{36 - 5y + y}{36 - 5y - y} = \frac{5}{3}$

$\frac{36 - 4y}{36 - 6y} = \frac{5}{3}$

$108 - 12y = 180 - 30y$

$18y = 72$

$y = 4$

$x = 36 - 5y$

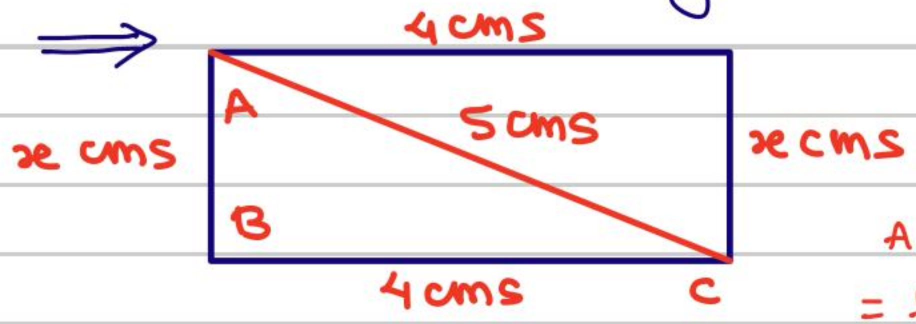
$x = 36 - 5(4)$

$x = 16$

$(x, y) \equiv (16, 4)$

(157) Diagonal of a rectangle is 5 cms and one of the side is 4 cms then Find Area of Rectangle.

$\Longrightarrow$



$5^2 = 4^2 + x^2$

$x^2 = 25 - 16 = 9$

$\therefore x = 3$ cms.

Area of Rectangle

= length × Breadth

= 4 cms × 3 cms

= 12 cm^2 OR 12 sq.cms

(158) If one root of quadratic equation exceeds the other by 4 in $x^2 - 8x + m = 0$. Find m.

$\Longrightarrow$ $x^2 - 8x + m = 0$

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$x^2 - (2 + 6)x + (2 \times 6) = 0$

$x^2 - (2+6)x + 12 = 0$

$x^2 - 8x + 12 = 0$

$\therefore m = 12$

(159) $x + y = 50$, $\frac{1}{x} + \frac{1}{y} = \frac{1}{8}$ then $(x, y) = ?$

$\Longrightarrow$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{8}$

$\frac{x+y}{xy} = \frac{1}{8}$

$\frac{50}{xy} = \frac{1}{8}$

$xy = 400$

$x(50 - x) = 400$

$50x - x^2 = 400$

$x^2 - 50x + 400 = 0$

$(x-40)(x-10) = 0$

$x = 40$ OR $x = 10$

$y = 10$ OR $y = 40$

$(x, y) = (10, 40)$ OR $(40, 10)$

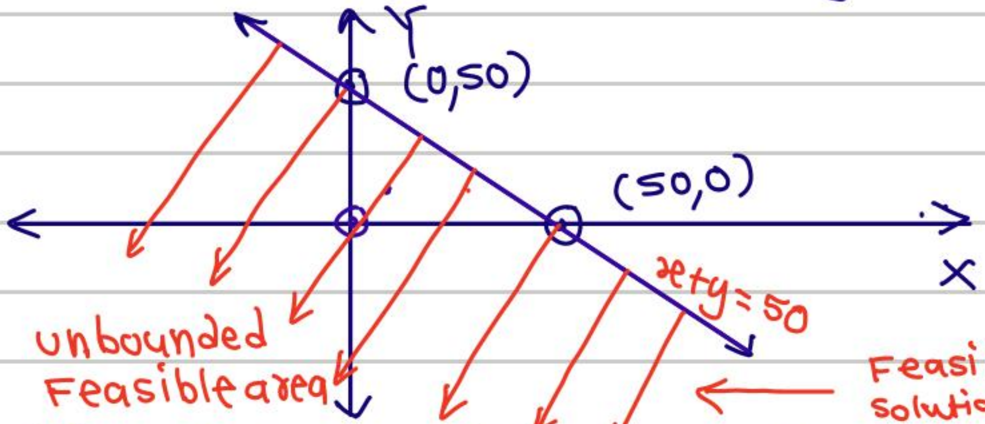
(160) Find Feasible area for $x+y \le 50$

$\Longrightarrow$

$x+y=50$ $2x+3y=90$ $3x-5y=60$ $x=35$	Linear equation OR Linear Equality	$x+y \le 50$ $2x-3y \ge 90$ $5x-18y < 35$ $x \le 48$ $y \ge 90$	Linear Inequations OR Linear Inequality

$x+y \le 50$ is a linear inequality.

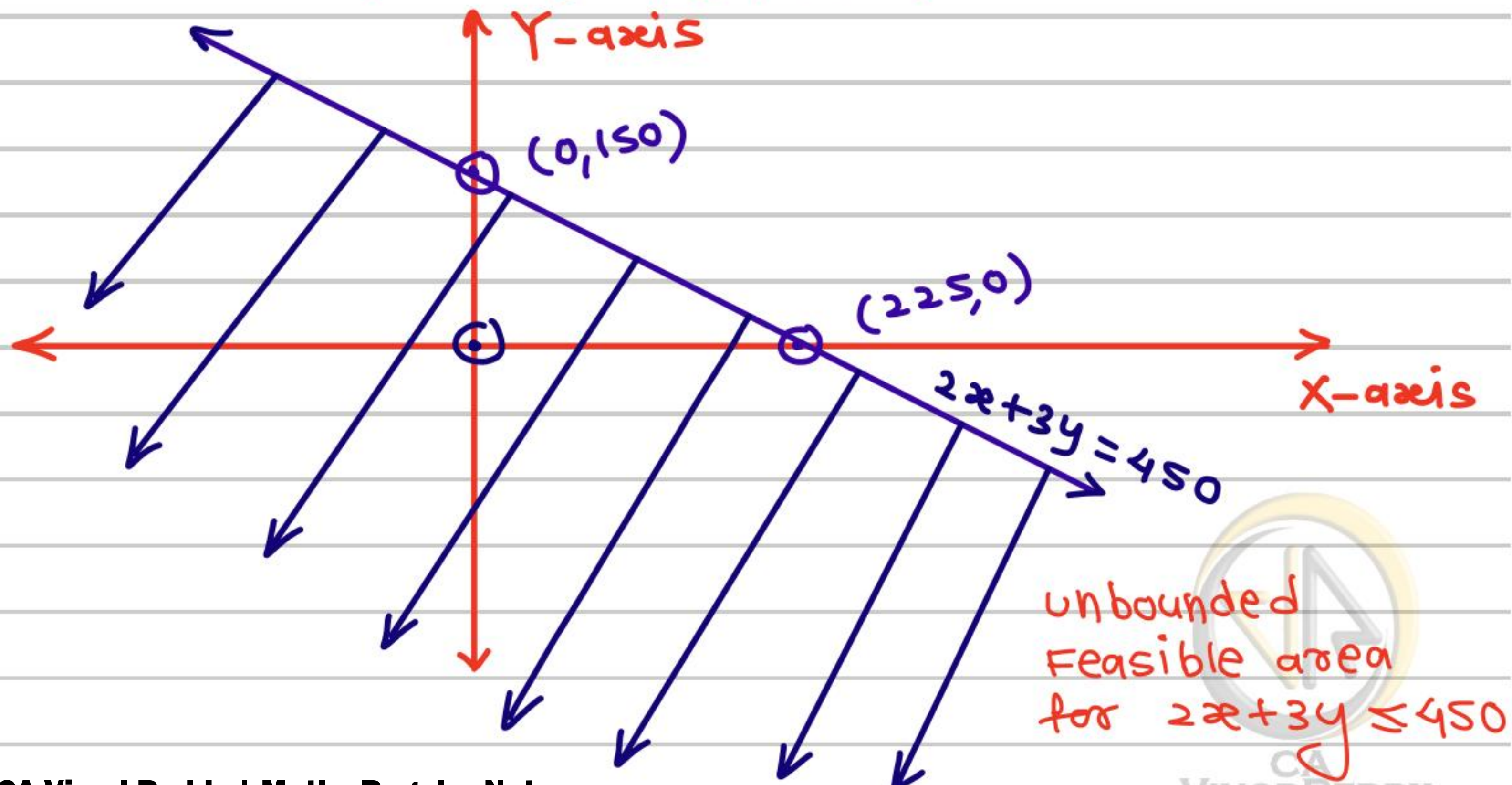
Let's Draw the line $x+y=50$ on graph paper



Graphical representation of a linear Equation is : LINE
Graphical representation of a linear inequality is : AN AREA

(161) Find Feasible area for $2x+3y \le 450$

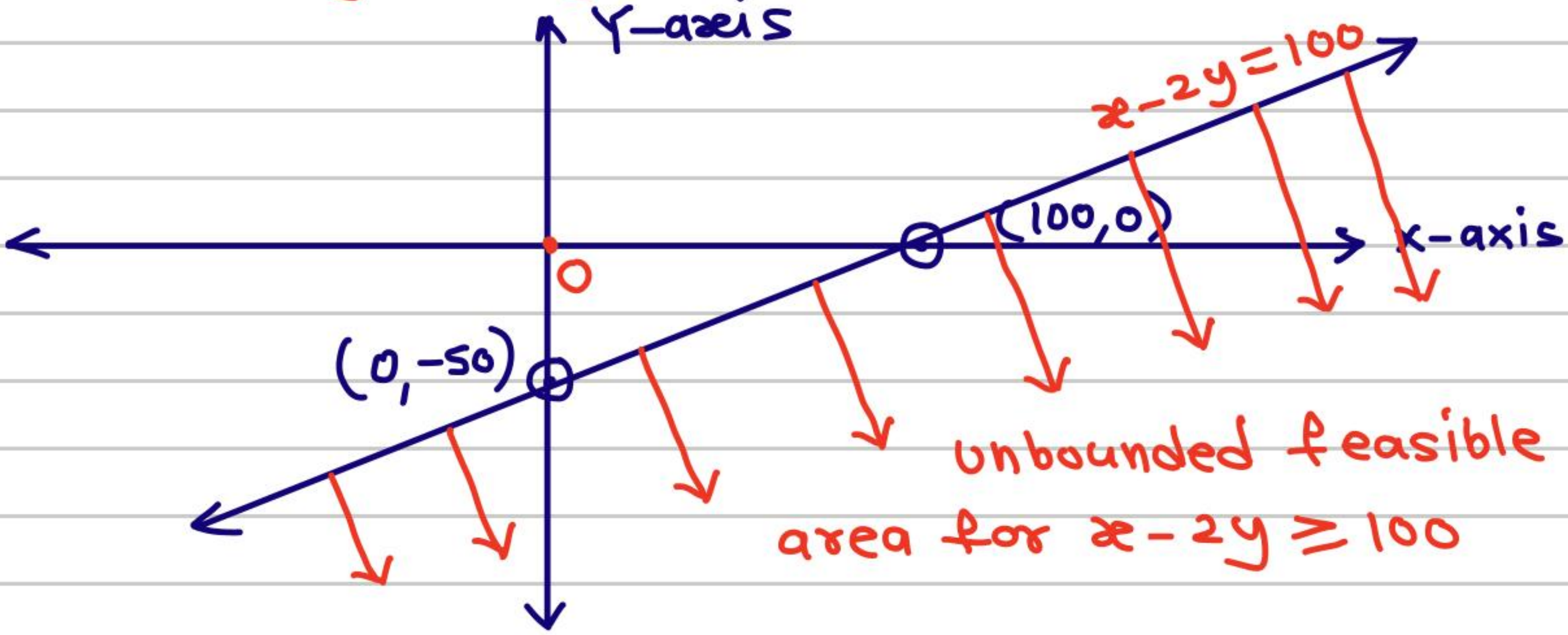
$\Longrightarrow$ To Find Feasible area for $2x+3y \le 450$ Let's draw the line $2x+3y=450$ by joining the points $(225,0)$ & $(0,150)$



(162) Find Feasible area for

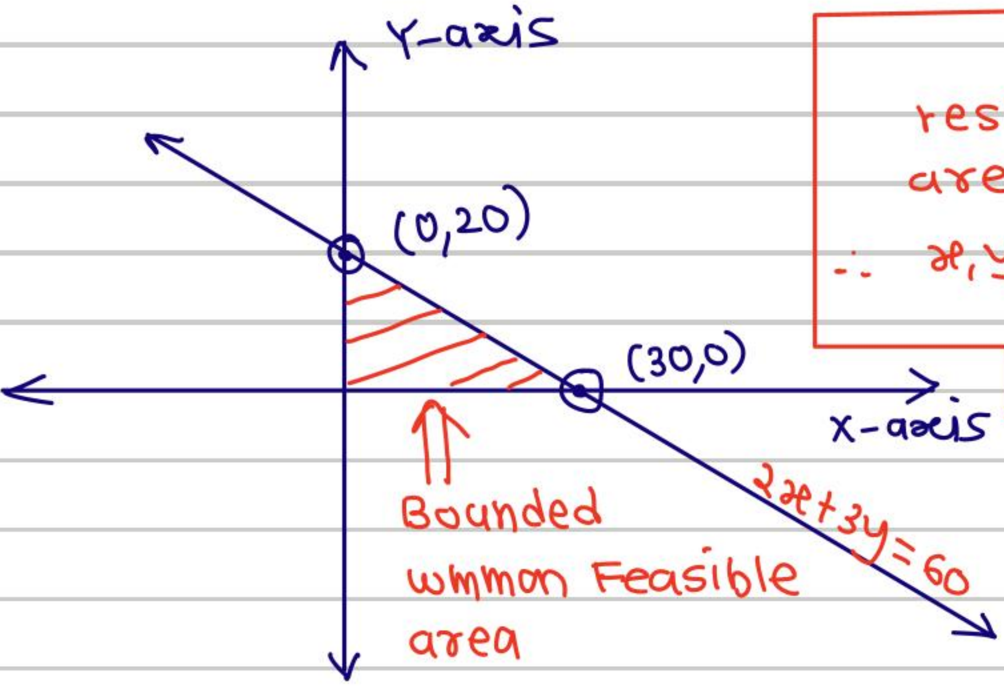
$x - 2y \geq 100$

$\Longrightarrow$ First we will draw the line $x - 2y = 100$ by joining points $(100,0)$ & $(0,-50)$



(163) Find common feasible area for $2x+3y \leq 60$ & $x, y \geq 0$

$\Longrightarrow$ Let's First draw $2x+3y=60$ by joining points $(0,20)$ & $(30,0)$



$x, y \geq 0$ will restrict the feasible area in 1st quadrant

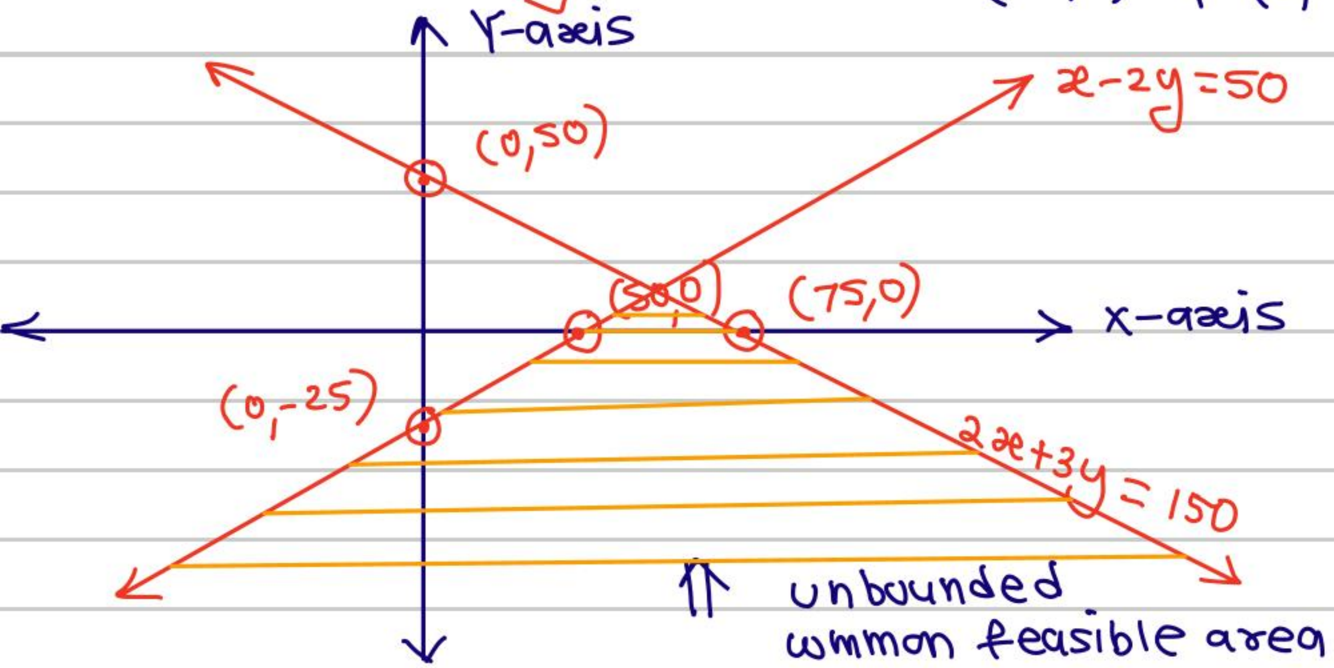
$\therefore$ $x, y \geq 0$ is known as Non-Negativity constraint

(164) Find common feasible area for

$x - 2y \geq 50$ & $2x + 3y \leq 150$

$\Longrightarrow$ $x - 2y = 50 \Longrightarrow (50,0)$ & $(0,-25)$

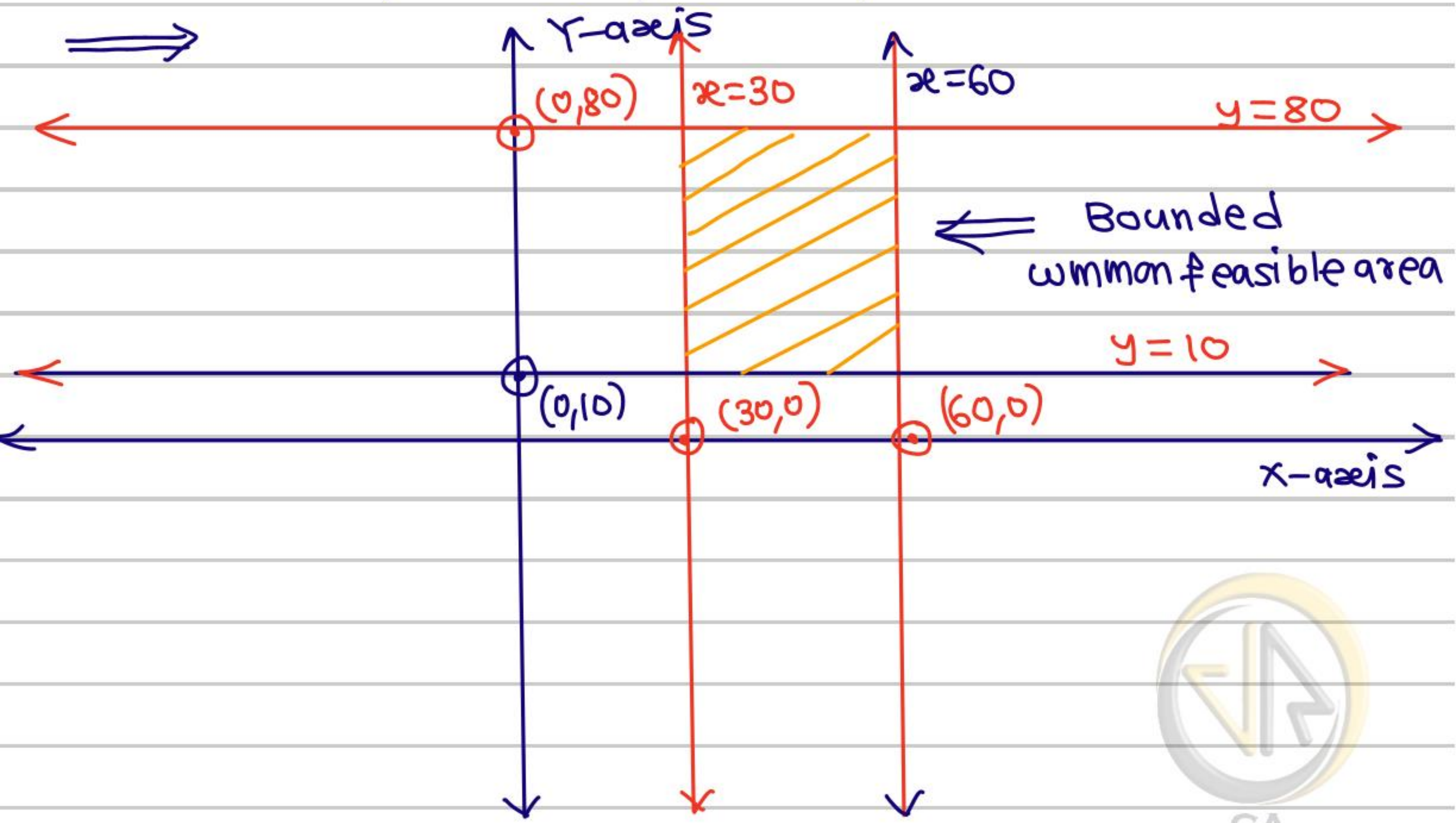
$2x + 3y = 150 \Longrightarrow (75,0)$ & $(0,50)$



(165) Find common feasible area for

$x \geq 30$, $x \leq 60$, $y \geq 10$, $y \leq 80$

$\Longrightarrow$

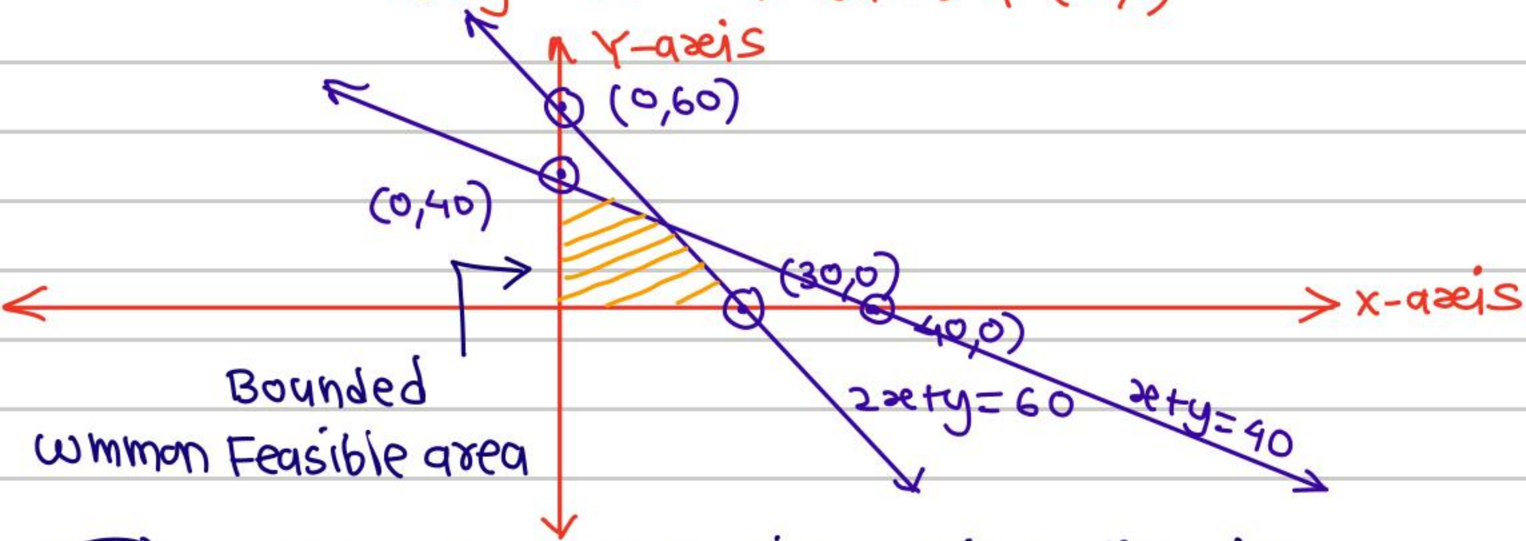


(166) Find common feasible area for

$2x + y \leq 60$, $x + y \leq 40$, $x, y \geq 0$

$\Longrightarrow$ $2x + y = 60 \Rightarrow (30,0)$ & $(0,60)$

$x + y = 40 \Rightarrow (0,40)$ & $(40,0)$



(167) Which of the following point lie in common feasible area of $5x - 3y \leq 80$, $2x + 7y \geq 40$

(a) $(5,2)$ (b) $(20,3)$ (c) $(30,30)$ (d) None

$\Longrightarrow$ $(5,2)$ $5(5) - 3(2) = 19 \leq 80$ ✓

$2(5) + 7(2) = 24 \geq 40$ ✗

$(20,3)$ $5(20) - 3(3) = 91 \leq 80$ ✗

$(30,30)$ $5(30) - 3(30) = 60 \leq 80$ ✓

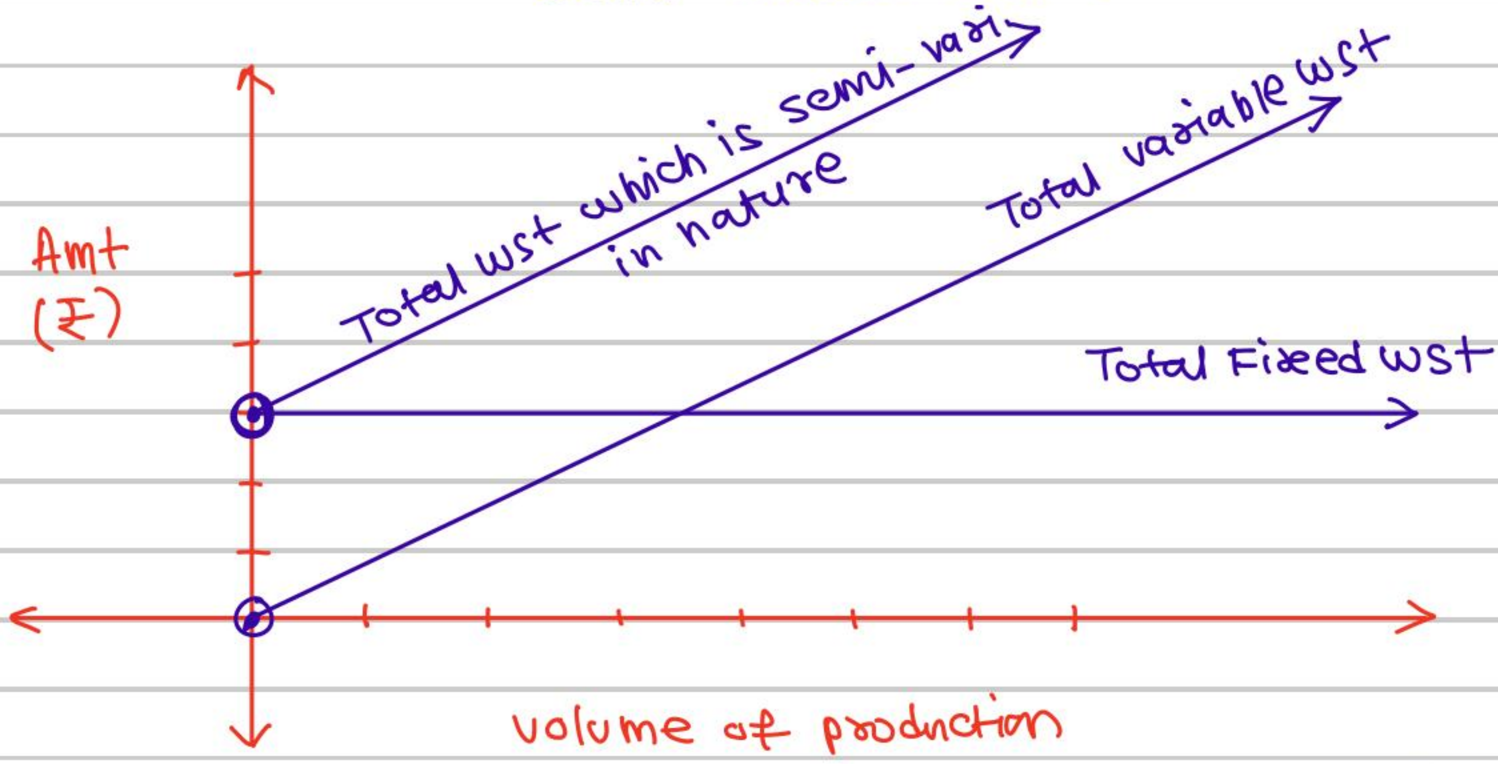
$2(30) + 7(30) = 270 \geq 40$ ✓

(168) Total wst = Fixed wst + Variable wst

Fixed wst : The wst which does not change with change in volume of production is known as Fixed wst

Variable wst : The wst which changes in <u>same proportion</u> with change in volume of production is known as variable wst

Semi-variable wst : If portion of the wst is Fixed and portion is variable then wst is said to be semi-variable OR semi-Fixed wst.



(169) If $b^2 - 4ab = 0$ then roots of quadratic Eqn are

(a) Equal (b) Equal but opposite in sign

(c) Reciprocals of each other

~~(d)~~ can't say

(170) Sum of 2 numbers is 88 and diff. betn first number and half of second number is 10. Find the Numbers

(a) 32, 56 (b) 44, 44 ~~(c)~~ 36, 52 (d) 30, 58

$\Longrightarrow$ $36 + 52 = 88$ OR $x + y = 88$

$36 - \frac{52}{2} = 10$ $x - \frac{y}{2} = 10$

(171) If p, q are roots of $3x^2 - 3x - 1 = 0$ Find value of $(p^3 + q^3)$, $(p^2 + q^2)$

$\Longrightarrow$ $p + q = -(-3)/3 = 1$

$pq = -1/3$

$$(p^3 + q^3) = (p+q)^3 - 3pq(p+q)$$
$$= 1^3 - \left(3 \times -\frac{1}{3} \times 1\right)$$
$$= 1 - (-1)$$
$$= 1 + 1$$
$$= 2$$

$$p^2 + q^2$$
$$= (p+q)^2 - 2pq$$
$$= 1 - 2 \times \left(-\frac{1}{3}\right)$$
$$= 1 + \frac{2}{3}$$
$$= 5/3$$

(172) If p, q are roots of $3x^2 - 19x - 1 = 0$. Find quadratic Eqn whose roots are p/q & q/p

$\Longrightarrow$ $p+q = 19/3$, $pq = -1/3$

The quad. eqn whose roots are (p/q) & (q/p) is,

$$x^2 - \left(\frac{p}{q} + \frac{q}{p}\right)x + \left(\frac{p}{q} \times \frac{q}{p}\right) = 0$$

$$x^2 - \left(\frac{p^2+q^2}{pq}\right)x + 1 = 0$$

$$x^2 - \left[\frac{\frac{361}{9} + \frac{6}{9}}{-\frac{3}{9}}\right]x + 1 = 0$$

$$x^2 + \left(\frac{367/9}{3/9}\right)x + 1 = 0$$

$$x^2 + \frac{367}{3}x + 1 = 0$$

$$3x^2 + 367x + 3 = 0$$

(173) The cubic Eqn whose roots are m, n, q is :

$\Longrightarrow$ $x^3 - (m+n+q)x^2 + (mn + nq + mq)x - (mnq) = 0$

(174) If x = No. of units produced

Fixed cost = ₹3,80,000

variable cost p.u. = ₹28

then y = Total cost = ________

$\Longrightarrow$ Total cost = Fixed cost + variable cost

Total cost = Fixed cost + (vari. cost p.u. × No. of units produced)

$$\boxed{y = 3{,}80{,}000 + 28x}$$

(175) If $(p+2)(p-3)+(p+3)(p-4)=p(2p-5)$ then $p=?$

$\Longrightarrow$ $(p+2)(p-3)+(p+3)(p-4)=p(2p-5)$

$p^2-3p+2p-6+p^2-4p+3p-12=2p^2-5p$

$2p^2-2p-18=2p^2-5p$

$\cancel{2p^2}-2p-18-\cancel{2p^2}+5p=0$

$3p-18=0$

$3p=18$

$p=18/3$

$p=6$

(176) $15x+23y=-10$ &

$3x+4y=-2$ then $3x+2y+2=?$

$\Longrightarrow$

$\cancel{15x}+23y=-10$

$\cancel{15x}+20y=-10$

$-\quad\quad -\quad\quad +$

$3y=0$

$\boxed{y=0}$

$15x+23y=-10$

$15x+23(0)=-10$

$15x=-10$

$x=-10/15$

$\boxed{x=-2/3}$

$3x+2y+2$

$=3\left(-\frac{2}{3}\right)+2(0)+2$

$=-2+0+2$

$=0$

(177) Find value of k, If $9x^2 - 24x + k = 0$ has equal roots.

$\Longrightarrow$ As roots of quadratic Eq^n are equal,

$$b^2 - 4ac = 0$$

$$(-24)^2 - 4(9)(k) = 0$$

$$576 - 36k = 0$$

$$576 = 36k$$

$$\therefore \; k = 16$$

(178) calculate the number such that It is equal to 3 times of its diff from 56.

$\Longrightarrow$ $x = 3 \times (56 - x)$

$$x = 168 - 3x$$

$$4x = 168$$

$$\boxed{x = 42}$$

(179) $2x + 3y = 5$

$3x - 4y = 2$ then $5xy = ?$

$\Longrightarrow$

$$\cancel{6x} + 9y = 15$$

$$\cancel{6x} - 8y = 4$$

$$- \quad + \quad -$$

$$17y = 11$$

$$\boxed{y = 11/17}$$

$$2x + 3 \times \frac{11}{17} = 5$$

$$2x = 5 - \frac{33}{17}$$

$$2x = \frac{52}{17}$$

$$\therefore \boxed{x = \frac{26}{17}}$$

$$5xy = 5 \times \frac{26}{17} \times \frac{11}{17}$$

$$= \left(\frac{1430}{289}\right)$$

(180) $a^2 + b^2 = 45$ then $\left(\frac{1}{a} + \frac{1}{b}\right) = ?$

$ab = 18$

$\Longrightarrow$

$(a+b)^2 = (a^2+b^2) + (2ab)$

$(a+b)^2 = 45 + (2 \times 18)$

$(a+b)^2 = 45 + 36 = 81$

$\therefore (a+b) = 9$

$\frac{1}{a} + \frac{1}{b}$

$= \left(\frac{a+b}{ab}\right) = \left(\frac{9}{18}\right)$

$= \frac{1}{2} = 0.50$

(181) If roots of quadratic eqn are $(2m)$ & $(-2n)$ then Factors are:

$\Longrightarrow$ $(x-2m), (x+2n)$

(182) If roots of quadratic Eqn are $3/5$ & $-8/111$ then factors are:

$\Longrightarrow$ $(5x-3), (111x+8)$

(183) If quadratic equation

$x^2 - (p+4)x + 2p+5 = 0$ has equal roots.

Find p.

$\Longrightarrow \left[-(p+4)\right]^2 - 4(1)(2p+5) = 0$

$p^2 + \cancel{8p} + 16 - \cancel{8p} - 20 = 0$

$p^2 - 4 = 0$

$p^2 - 2^2 = 0$

$(p-2)(p+2) = 0$

$\therefore \quad p = 2 \quad \text{OR} \quad p = -2$

(184) If $4x^3 + 8x^2 - x - 2 = 0$ then $(2x+3) = ?$

~~(a)~~ 4, -1, 2 (b) -4, 2, 1 (c) 2, -4, -1 (d) None

$\Longrightarrow$

$4x^3 + 8x^2 - x - 2 = 0$

$4x^2(x+2) - 1(x+2) = 0$

$(x+2)(4x^2 - 1) = 0$

$(x+2)\left[(2x)^2 - 1^2\right] = 0$

$(x+2)(2x-1)(2x+1) = 0$

$x = -2$ OR $x = \frac{1}{2}$ OR $x = -\frac{1}{2}$

x	$2x+3$
-2	$2(-2)+3$ $= -1$
$\frac{1}{2}$	$2\left(\frac{1}{2}\right)+3$ $= 4$
$-\frac{1}{2}$	$2\left(-\frac{1}{2}\right)+3$ $= -1+3$ $= 2$

(185) sum of 2 numbers is 15 & their product is 50 then sum of their reciprocals is :

$\Longrightarrow$ $x+y=15$

$xy=50$

$\frac{1}{x}+\frac{1}{y}$

$=\frac{x+y}{xy}=\frac{15}{50}=3/10$

$=0.30$

(186) out of 3 numbers, sum of first and second is 24, sum of 2nd & 3rd is 30, sum of first & Third is 26. The smallest Number is :

(a) 18 (b) 14 (c) 16 ~~(d)~~ 10

$\Longrightarrow$ Let 3 numbers be x, y, z

$x+y=24$ ----- ①

$y+z=30$ ---- ②

$x+z=26$ ---- ③

$x+y=24$

$x+30-z=24$

$x+30-(26-x)=24$

$x+30-26+x=24$

$2x+4=24$

$2x=20$

$\boxed{x=10}$

$x=10, y=14, z=16$

$\therefore$ 3 Numbers are : 10, 14, 16

& smallest Number is 10